Vesic's Bearing-Capacity Equations

- The Vesic procedure is essentially the same as the method of Hansen (1961) with select changes.

$$q_{ult} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5B'N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

N_q =same as Meyerhof above.

N_c =same as Meyerhof above.

$$N_\gamma = 2(N_q + 1)\tan\phi$$

- The N_c and N_q terms are those of Hansen but N_γ is slightly different, see Table 1-6.
- There are also differences in the i_i, $b_{i,}$ and g_i terms as in Table 1-7.
- The Vesic equation is somewhat easier to use than Hansen's because Hansen uses the i terms in computing shape factors s_i whereas Vesic does not.

Table 1-6 Bearing-capacity factors for Vesic's bearing-capacity equation

ϕ	N_c	N_q	N_γ
0	5.14	1.00	0.00
1	5.38	1.09	0.07
2	5.63	1.20	0.15
3	5.90	1.31	0.24
4	6.19	1.43	0.34
5	6.49	1.57	0.45
6	6.81	1.72	0.57
7	7.16	1.88	0.71
8	7.53	2.06	0.86
9	7.92	2.25	1.03
10	8.34	2.47	1.22
11	8.80	2.71	1.44
12	9.28	2.97	1.69
13	9.81	3.26	1.97
14	10.37	3.59	2.29
15	10.98	3.94	2.65
16	11.63	4.34	3.06
17	12.34	4.77	3.53
18	13.10	5.26	4.07
19	13.93	5.80	4.68
20	14.83	6.40	5.39
21	15.81	7.07	6.20
22	16.88	7.82	7.13
23	18.05	8.66	8.20
24	19.32	9.60	9.44
25	20.72	10.66	10.88

ϕ	N_c	N_q	N_γ
26	22.25	11.85	12.54
27	23.94	13.20	14.47
28	25.80	14.72	16.72
29	27.86	16.44	19.34
30	30.14	18.40	22.40
31	32.67	20.63	25.99
32	35.49	23.18	30.21
33	38.64	26.09	35.19
34	42.16	29.44	41.06
35	46.12	33.30	48.03
36	50.59	37.75	56.31
37	55.63	42.92	66.19
38	61.35	48.93	78.02
39	67.87	55.96	92.25
40	75.31	64.20	109.41
41	83.86	73.90	130.21
42	93.71	85.37	155.54
43	105.11	99.01	186.53
44	118.37	115.31	224.63
45	133.87	134.87	271.75
46	152.10	158.50	330.34
47	173.64	187.21	403.65
48	199.26	222.30	496.00
49	229.92	265.50	613.14
50	266.88	319.06	762.86

TABLE (1-7) Shape, depth, inclination, ground, and base factors for use in the Vesic bearing-capacity equations.

Factor	Value	Note
Shape factor	$s_c = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L}$ $s_c = 1.0$ for strip $s_q = 1.0 + \frac{B}{L} \tan\phi$ $s_\gamma = 1 - 0.4\frac{B}{L} \quad (\geq 0.6)$	
Depth factor	$d_c = 0.4k \quad (for\ \phi = 0)$ $d_c = 1.0 + 0.4k$ $d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2 k$ $d_\gamma = 1.0$	$k = D/B \quad (for\ D/B \leq 1)$ $k = \tan^{-1}(D/B)\ (for\ D/B > 1)$ k in radian
Inclination factor	$i_c = 1 - \frac{mH_i}{A_f C_a N_c} \quad (for\ \phi = 0)$ $i_c = i_q - \frac{1 - i_q}{N_q - 1} \quad (for\ \phi > 0)$ $i_q = \left[1 - \frac{H_i}{V + A_f C_a \cot\phi}\right]^m$ $i_\gamma = \left[1 - \frac{H_i}{V + A_f C_a \cot\phi}\right]^{m+1}$	$m = m_B = \frac{2 + B/L}{1 + B/L}$ $m = m_L = \frac{2 + L/B}{1 + L/B}$
Ground factor (base on slope)	$g_c = \frac{\beta}{5.14} \quad (for\ \phi = 0)$ $g_c = i_q - \frac{1 - i_q}{5.14\tan\phi} \quad (for\ \phi > 0)$ $g_q = g_\gamma = (1 - \tan\beta)^2$	β= in radians
Base factor (tilted base)	$b_c = g_c \quad (for\ \phi = 0)$ $b_c = 1 - \frac{2\beta}{5.14\tan\phi} \quad (for\ \phi > 0)$ $b_q = b_\gamma = (1 - \eta\tan\phi)^2$	η in radians

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2\sin(\pm\beta)$
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use the actual values of L and B not the effective values L' and B'.
3. Vesic always uses B' in the N_γ term of the bearing-capacity equation, (even when $H_i = H_L$).

Example 1-7

Re-compute the allowable bearing capacity of the case shown in Example 1-1 using Hansen's method.

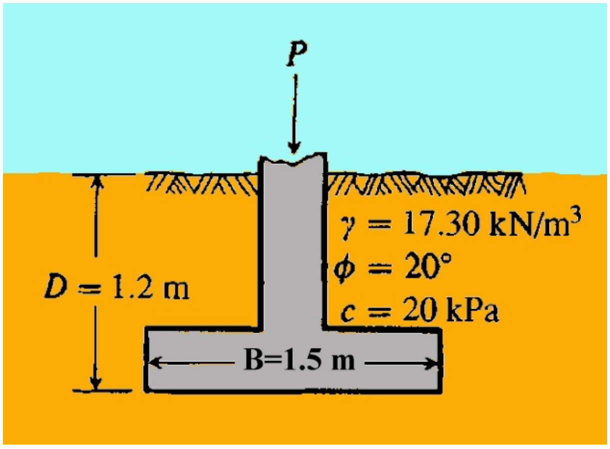


Figure 1-7 Soil properties and footing dimensions of Example 1-7

Solution

$$q_{ult} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5B'N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

Bearing capacity factors

From table 1-5 (for $\phi = 20°$), we obtain

$$N_c = 14.83, N_q = 6.4 \text{ and } N_\gamma = 5.39$$

Shape factors

$$s_c = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L} = 1.0 + \frac{6.4}{14.83} \cdot \frac{1.5}{1.5} = 1.431$$

$$s_q = 1.0 + \frac{B}{L}\tan\phi = 1.0 + \frac{1.5}{1.5}\tan 20 = 1.36$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 0.6 \qquad (\geq 0.6)$$

Depth factors

$$k = D/B = \frac{1.2}{1.5} = 0.8 \quad (for\ D/B \leq 1)$$

$$d_c = 1 + 0.4\,k = 1.32$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2 k = 1.252$$

$$d_\gamma = 1.0$$

Substitute these values in the Vesic's bearing capacity Equation, we obtain

$$q_{ult} = 20(14.83)(1.431)(1.32) + 17.3(1.2)(6.4)(1.36)(1.252) \\ + 0.5(1.5)(5.39)(0.6)(1.0)$$

$$q_{ult} = 788.9\ kPa$$

For factor of safety, $SF = 3$

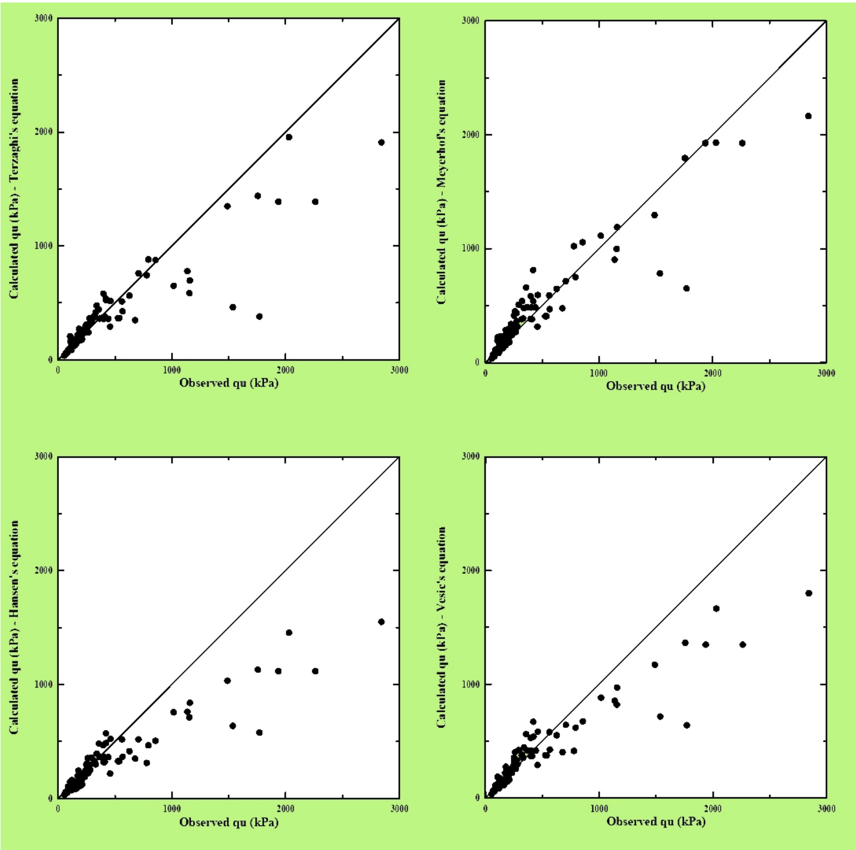
$$q_a = \frac{q_{ult}}{SF}$$

$$q_a = \frac{788.9}{3} \approx 263\ kPa$$

The value of q_a obtained here for Vesic's method is very close to the one obtained from Hansen's method.

Which Equation to use

- Comparing some measured values of q_{ult} obtained from experimental field tests shows that the computed q_{ult} to the measured values indicates none of the several theories/methods has a significant advantage over any other in terms of a best prediction.
- The four theoretical Equations have a reasonable accuracy, as indicated by field and laboratory tests shown below



- The Terzaghi equations, being the first proposed, have been very widely used. Both the Meyerhof and Hansen methods are widely used. The Vesic method has not been much used.

- From these observations one may suggest the following equation use:

Use	Best for
Terzaghi	Very cohesive soils where D/B < 1 or for a quick estimate of quit to compare with other methods. Do not use for footings with moments and/or horizontal forces or for tilted bases and/or sloping ground.
Hansen, Meyerhof, Vesic	Any situation that applies, depending on user preference or familiarity with a particular method.
Hansen, Vesic	When base is tilted; when footing is on a slope or when D/B > 1.

- It is good practice to use at least two methods and compare the computed values of q_{ult}. If the two values do not compare well, use a third method.

Footings with Eccentricity

Researches and observations indicate that effective footing dimensions obtained (refer to Fig. 1-8) as

$$L' = L - 2e_x \text{ and } B' = B - 2e_y$$

should be used in bearing-capacity analyses to obtain an effective footing area defined as

$$A_f = L'B'$$

and the center of pressure when using a rectangular pressure distribution of q' is the center of area BL' at point A'. From Fig 1-8:

$$2e_x + L' = L$$

$$e_x + c = L/2$$

Substitute for L and obtain $c = L'/2$. If there is no eccentricity about either axis, use the actual footing dimension for that B' or L'.

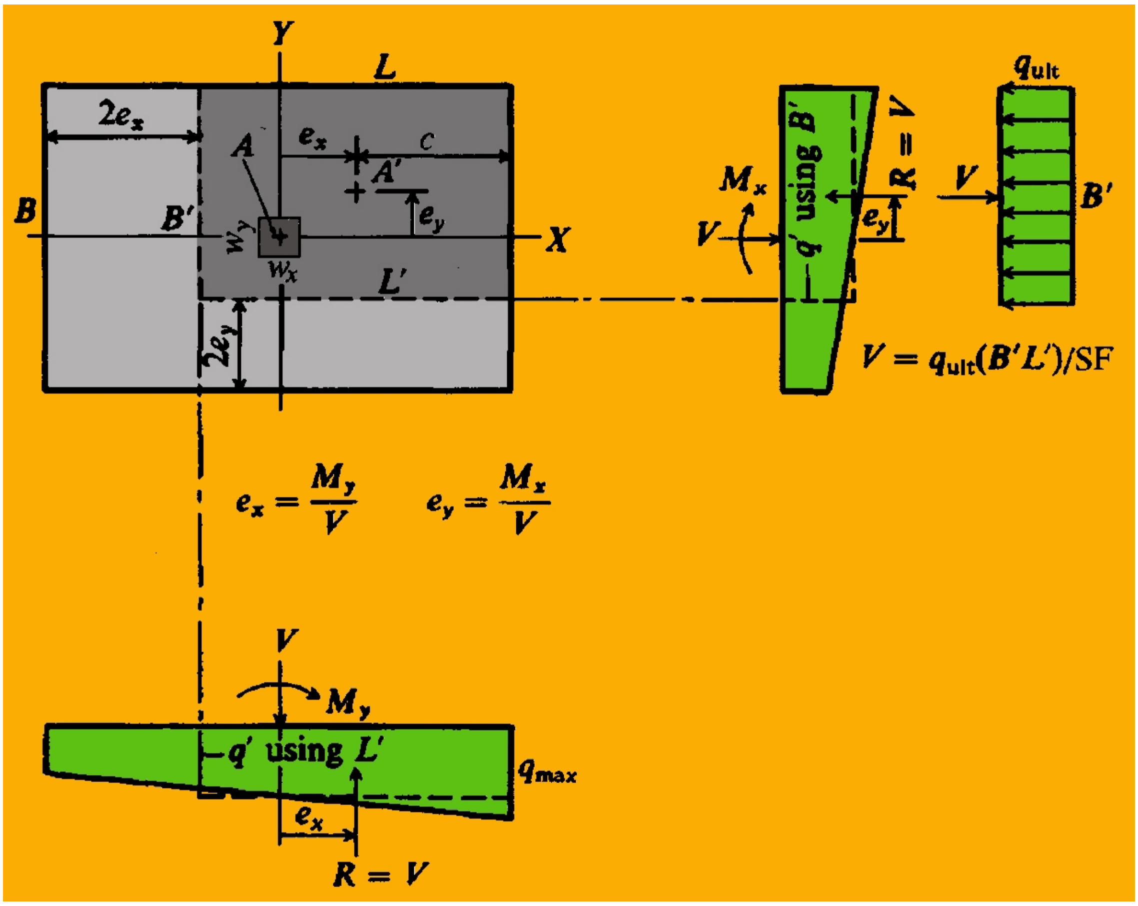


Figure 1-8 Method of computing effective footing dimensions when footing is eccentrically loaded for rectangular bases.

The effective area of a round base can be computed by locating the eccentricity e_x on any axis by swinging arcs with centers shown to produce an area *abcd*, which is then reduced to an equivalent rectangular base of dimensions $B' \times L'$ as shown on Fig. 1-9. You should locate the dimension B' so that the left edge (line c'd') is at least at the left face of the column located at point O.

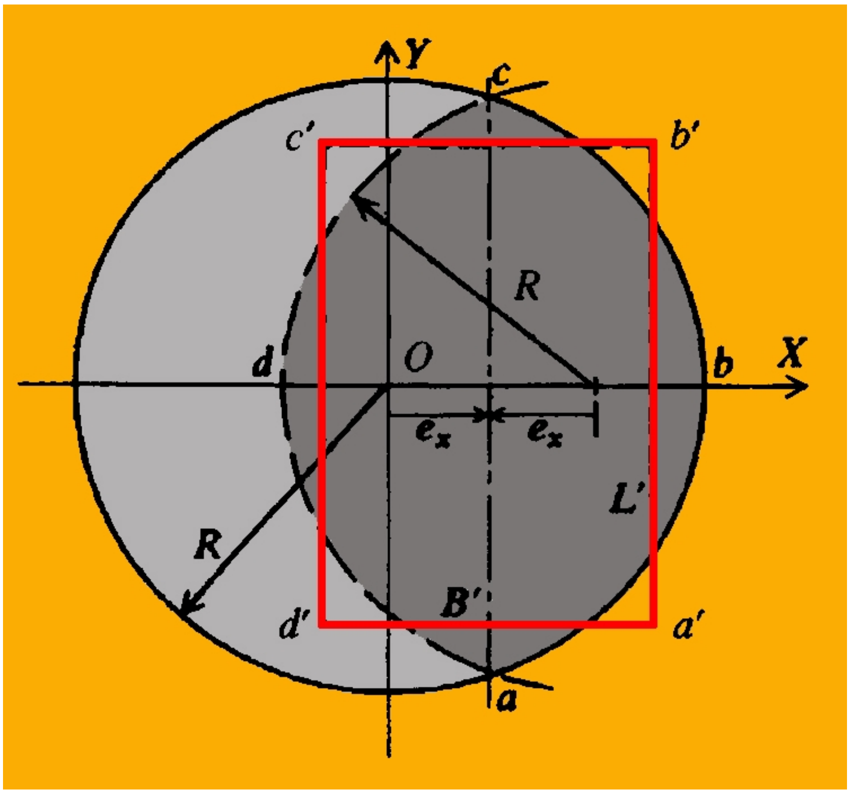


Figure 1-9 Method of computing effective footing dimensions when footing is eccentrically loaded for round bases.

For design the minimum dimensions (to satisfy ACI 318-) of a rectangular footing with a central column of dimensions $w_x \times w_y$ are required to be:

$$B_{min} = 4e_y + w_y \qquad B' = 2e_y + w_y$$

$$L_{min} = 4e_x + w_x \qquad L' = 2e_x + w_x$$

Final dimensions may be larger than B_{min} or L_{min} based on obtaining the required allowable bearing capacity.

The ultimate bearing capacity for footings with eccentricity, using either the Meyerhof or Hansen/Vesic equations, is found in either of two ways:

Method 1	Use either the Hansen or Vesic bearing-capacity equations with the following adjustments: a. Use B' in the $\gamma B N_\gamma$ term. b. Use B' and L' in computing the shape factors. c. Use actual B and L for all depth factors. The computed ultimate bearing capacity q_{ult} is then reduced to an allowable value q_a with an appropriate safety factor SF as $q_a = q_{ult}/SF$ and $P_a = q_a B'L'$
Method 2	Use the Meyerhof general bearing-capacity equation given in Table 4-1 and a reduction factor Re used as $q_{ult}^{eccentric} = q_{ult} \times R_e$ where: $q_{ult}^{eccentric}$ is the ultimate bearing capacity of footing subjected to eccentricity. R_e is the Meyerhof reduction factor (it is used only with the Meyerhof equation to compute the bearing capacity), and it is can be computed from $R_e = 1 - 2e/B$ (for cohesive soil) $R_e = 1 - \sqrt{e/B}$ (for cohesionless soil and for $0 < e/B < 0.3$)

- It should be evident from Fig. 1-8 that if $e/B = 0.5$, the point A' falls at the edge of the base and an unstable foundation results. In practice the e/B ratio is seldom greater than 0.2 and is usually limited to $e < B/6$.
- In these reduction factor equations the dimensions B and L are referenced to the axis about which the base moment occurs.

Normally, greatest base efficiency is obtained by using the larger or length dimension L to resist overturning.

- For round bases use B as the diameter.
- Alternatively, one may directly use the Meyerhof equation with B' and L' used to compute the shape and depth factors and B' used in the $0.5\gamma B' N_\gamma$ term.

Example 1-8.

A square footing is 1.8 X 1.8 m with a 0.4 X 0.4 m square column. It is loaded with an axial load of 1800 kN and M_x= 450 kN.m; M_y = 360 kN.m. Undrained triaxial tests (soil not saturated) give ϕ = 36° and c = 20 kPa. The footing depth D = 1.8 m; the soil unit weight

γ = 18.00 kN/m³.

Required. What is the allowable soil pressure, if SF = 3.0, using the Hansen and Meyerhof bearing-capacity equations.

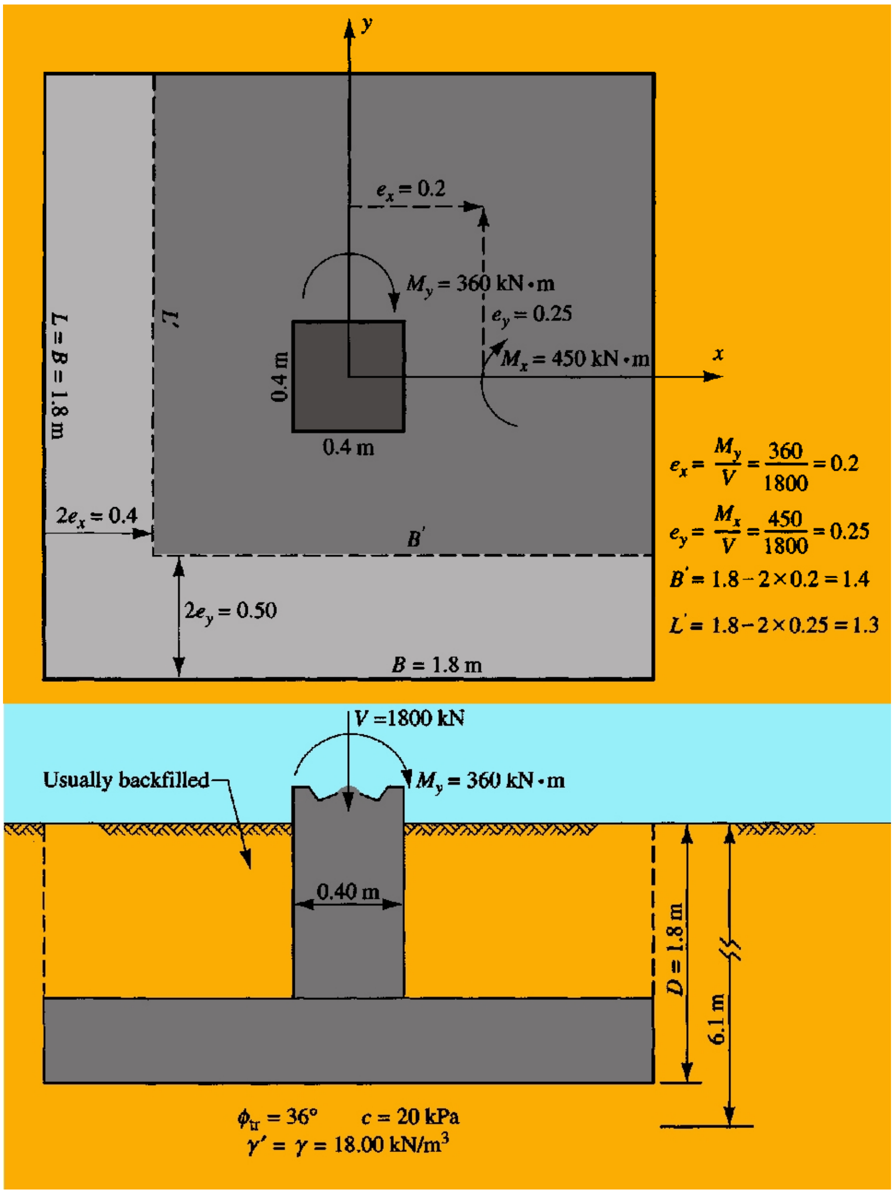


Figure 1-10 Illustration of Example 1-8

Solution: see Fig. 1-10

$$e_y = \frac{450}{1800} = 0.25\ m$$

$$e_x = \frac{360}{1800} = 0.20\ m$$

Both values of e are $< B/6 = 1.8/6 = 0.30\ m$. Also

$$B_{min} = 4(0.25) + 0.4 = 1.4 < 1.8\ m \quad \text{(available width)}$$

$$L_{min} = 4(0.20) + 0.4 = 1.2 < 1.8\ m \quad \text{(available length)}$$

Now find

$$B' = B - 2e_y = 1.8 - 2(0.25) = 1.3\ m$$

$$L' = L - 2e_x = 1.8 - 2(0.20) = 1.4\ m$$

By Hansen's Equation:

$$q_{ult} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5\gamma B'N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

Bearing capacity factors

From Table 1-4 at $\phi = 36°$

$N_c = 50.59 \quad N_q = 37.75 \quad N_\gamma = 40.05$

Shape factors (use $B = B'$ and $L = L'$)

$$s_c = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L} = 1.0 + \frac{37.75}{50.59} \cdot \frac{1.3}{1.4} \rightarrow s_c = 1.69$$

$$s_q = 1.0 + \frac{B}{L} \sin\phi = 1.0 + \frac{1.3}{1.4} \sin 36 \rightarrow s_q = 1.55$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4\frac{1.3}{1.4} \rightarrow s_\gamma = 0.63 \quad (\geq 0.6)$$

Depth factors (use B and L)

$k = D/B \quad (for\ D/B \leq 1), \rightarrow k = \frac{1.8}{1.8} = 1.0$

$d_c = 1.0 + 0.4k \rightarrow d_c = 1.4$

$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2 k = 1 + 2\tan 36\,(1 - \sin 36)^2 \times (1.0) \rightarrow d_q = 1.25$

$d_\gamma = 1.0$

Inserting values computed in the bearing capacity Equation ((**use $B = B'$**)

$q_{ult} = 20(50.59)(1.69)(1.4) + 1.8(18.0)(37.75)(1.55)(1.25) + 0.5(18.0)(1.3)(40.05)(0.63)(1.0)$

$$\rightarrow q_{ult} = 5058\ kPa$$

For factor of safety, $SF = 3$

$$q_a = \frac{q_{ult}}{SF}$$

$$q_a = \frac{5058}{3} = 1686\ kPa$$

The actual soil pressure is

$$q_{act} = \frac{P}{B'L'} = \frac{1800}{1.3 \times 1.4} = 989\ kPa$$

By Meyerhof's method and R_e (This method uses actual base dimension B and L)

For vertical load: $q_{ult} = cN_cS_cd_c + qN_qS_qd_q + 0.5\gamma B'N_\gamma S_\gamma d_\gamma$

Bearing capacity factors

From Table 1-2 at $\phi = 36°$

$N_c = 50.59 \quad N_q = 37.75 \quad N_\gamma = 44.43$

Shape factors

From Table 1-3

First, compute the value of K_p,

$K_p = \tan^2(45 + \phi/2) = \tan^2(45 + 36/2)$=3.85

$s_c = 1 + 0.2K_p \frac{B}{L}$	→	$s_c = 1 + (0.2)(3.85)\frac{1.8}{1.8}$	→	$s_c = 1.77$
$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$	→	$s_q = s_\gamma = 1 + (0.1)(3.85)\frac{1.8}{1.8}$	→	$s_q = s_\gamma =$ 1.39

Depth factors

From Table 1-3

$d_c = 1 + \sqrt{K_p}\frac{D}{B}$	→	$d_c = 1 + \sqrt{3.85}\frac{1.8}{1.8}$	→	$d_c = 1.39$
$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	→	$d_q = d_\gamma = 1 + 0.1\sqrt{3.85}\frac{1.8}{1.8}$	→	$d_q = d_\gamma =$ 1.20

$$q_{ult} = (20)(50.59)(1.77)(1.39) + 1.8(18)(37.75)(1.39)(1.20) + 0.5(18)(1.8)(44.43)(1.39)(1.20)$$

$$q_{ult} = 5730\ kPa$$

There will be two reduction factors since there is two-way eccentricity.

$$R_{eB} = 1 - \frac{2e_y}{B} = 1 - \frac{2(0.25)}{1.8} = 0.72$$

$$R_{eL} = 1 - \frac{2e_x}{B} = 1 - \frac{2(0.20)}{1.8} = 0.78$$

$$q_{ult} = 5730 \times R_{eB} \times R_{eL} = 5730 * 0.72 * 0.78 \rightarrow q_{ult} = 3218\ kPa$$

For factor of safety,$SF = 3$

$$q_a = \frac{q_{ult}}{SF}$$

$$q_a = \frac{3218}{3} \approx 1072\ kPa$$