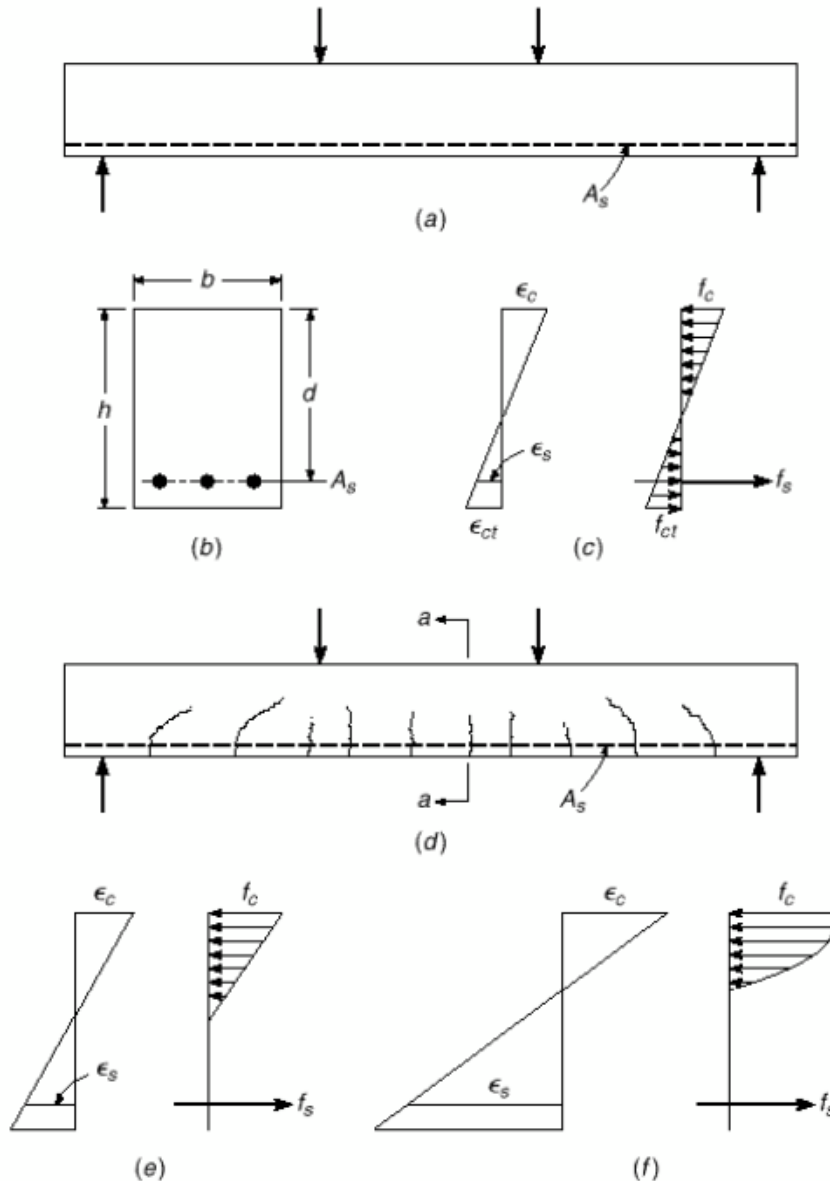


2 Flexural Analysis and Design of RC Beams

RC Beam Behavior



1st stage (fig. c): At low loads, all stresses are of small magnitude and are proportional to strains.

2nd stage (fig. e): When the load is increased, the tensile strength of concrete is reached; tension cracks develop; concrete does not transmit any tensile stresses. The steel resists the entire tension. If concrete compressive stresses do not exceed $\approx 0.5 f'_c$, stresses and strains continue to be proportional (linear stress distribution)

3rd stage (fig. f): When the load is further increased, stresses and strains are no longer proportional; the distribution of concrete stresses on the compression side is of the same shape as concrete stress-strain curve.

Failure can be caused in one of two ways:

A) **When moderate amounts of reinforcement are employed**, the steel will reach its yield point; the reinforcement stretches a large amount; the tension cracks widen and propagate upward; significant deflection of the beam; When this happens, the strains in compression zone of concrete increase to ensue crushing (*secondary compression failure*) at a load only slightly greater than that which cause the steel to yield.
Such yield failure is gradual and is preceded by visible signs of distress: cracks and deflection.

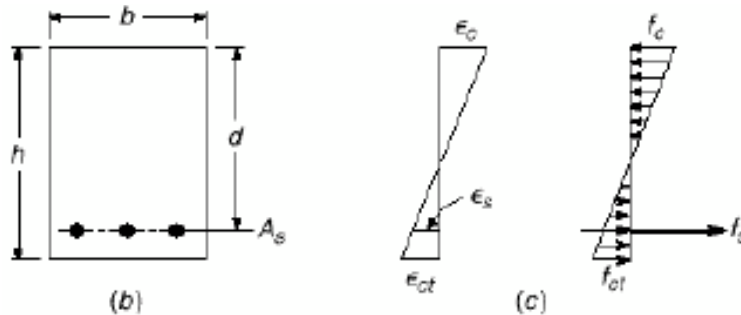
B) **When large amounts of reinforcement are employed**, the compressive strength of concrete is exhausted before the steel starts yielding. It has been observed that beams fail in compression when the concrete strains reach values of about 0.003 to 0.004.
Such compression failure is sudden; explosive, and occurs without warning.

It is good practice to dimension beams that they will fail by yielding of the steel (A) rather than by crushing of concrete (B).

Analysis of Stresses and Strength in the Different Stages

a) Stresses Elastic and Section Uncracked (figure c)

Tensile stresses are less than the modulus of rupture f_r .



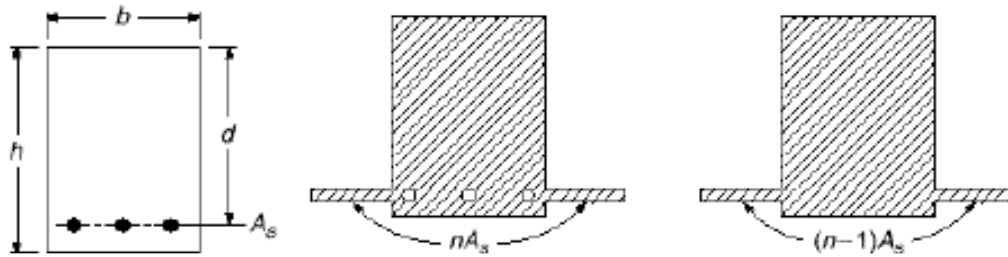
At the level of reinforcement:

$$\begin{aligned}\epsilon_s &= \epsilon_c ; \\ f_s / E_s &= f_c / E_c ; \\ f_s &= (E_s / E_c) f_c \\ f_s &= n f_c\end{aligned}$$

Where $n = E_s / E_c$ is known as the *modular ratio*.

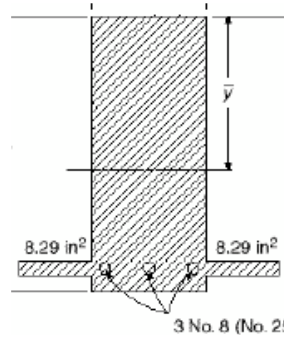
It means that the stress in steel ($f_s = n f_c$) is n times that of the concrete. The analysis shall depend on the “transformed section”.

In this fictitious section, the actual area of the reinforcement is replaced with an equivalent concrete area equal to nA_s , located at the level of steel. (Figure below)



Example 1

A rectangular beam has the dimensions: $b = 250$ mm, $h = 650$ mm, and $d = 600$ mm and is reinforced with 3 No. 25 bars so that $A_s = 1530$ mm². The concrete cylinder strength f'_c is 28 MPa, and the tensile strength in bending (modulus of rupture) f_r is 3.27 MPa. The yield point of the steel f_y is 420 MPa. Determine the stresses caused by a bending moment $M = 61$ kN-m.



Solution

$$E_c = 4700 \sqrt{f'_c} = 24870 \text{ MPa}$$

$$n = E_s / E_c = 200000 / 24870 = 8$$

$$\text{Add an area } (n - 1)A_s = 7 \times 1530$$

$$= 10710 \text{ mm}^2 \{ 5355 \text{ mm}^2 (8.29 \text{ in}^2) \text{ as shown} \}$$

$$y^- = \sum A.y / \sum A = 342 \text{ mm from top (check this)}$$

$$I = 6,481,000,000 \text{ mm}^4 \text{ (check this)}$$

$$\text{Compression stress at top } f_c = M.y^- / I$$

$$= 61,000,000 \times 342 / 6,481,000,000 = \underline{3.22 \text{ MPa}}$$

$$\text{Tension stress at bottom } f_{ct} = 61,000,000 \times 308 / 6,481,000,000 = \underline{2.90 \text{ MPa}}$$

Since $2.90 \text{ MPa} < f_r = 3.27 \text{ MPa}$ (given), no tensile cracks will form, and calculation by uncracked transformed section is justified.

$$\text{Steel stress } f_s = n M.y / I$$

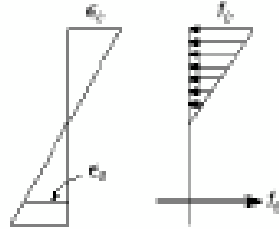
$$= 8 (61,000,000 \times 258 / 6,481,000,000) = \underline{19.43 \text{ MPa}}$$

It is seen that at this stage the actual stresses are quite small compared with the available strengths of steel and concrete.

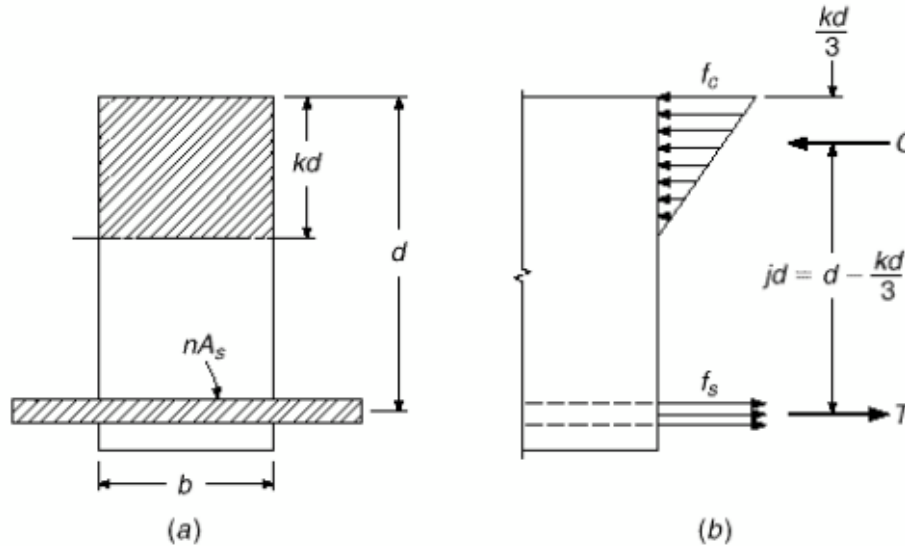
b) Stresses Elastic and Section Cracked (figure e)

When the tensile stress f_{ct} exceeds the modulus of rupture f_r , cracks form. If the compressive stress f_c is less than $\approx 0.5 f'_c$ and the steel stress has not reached the yield point ($f_s < f_y$), both materials continue to behave elastically.

This situation occurs in structures under normal service conditions and loads. This situation with regard to strain and stress distribution is that shown in figure e:



The fact is that all of the concrete that is stressed in tension is assumed cracked, and therefore effectively absent. (Figure below: cracked transformed section)



To determine the location of the N.A. (kd from top), the moment of the tension area about the axis is set equal to the moment of the compression area:

$$b(kd)^2/2 - nA_s(d - kd) = 0 \quad \dots\dots(1)$$

Then determine the moment of inertia.

Alternatively,

$$C = (f_c / 2) b(kd) \quad \text{and} \quad T = A_s f_s$$

$$M = C(jd) = (f_c / 2) b(kd) (jd) \quad = \quad M = T(jd) = A_s f_s (jd) \quad [Equilibrium]$$

$$f_c = 2M / (k j b d^2) \quad \text{and} \quad f_s = M / (A_s j d)$$

Definition: Reinforcement ratio:

$$\rho = A_s / bd$$

It is convenient to find k and j directly to establish NA distance: kd
 substituting $A_s = \rho bd$ into equation (1), solving for k

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

then $jd = d - kd/3$,

$$j = 1 - k/3$$

Values of k and j are tabulated (Nilson Table A6):

Example 2

The beam of Example 1 is subjected to a bending moment $M = 122 \text{ kN-m}$ (rather than 61 kN-m). Calculate the relevant properties and stresses.

Solution

Check the section is cracked:

Tension stress at bottom $f_{ct} = 122,000,000 \times 308 / 6,481,000,000 = 5.8 \text{ MPa}$

Since $5.8 \text{ MPa} > f_r = 3.27 \text{ MPa}$ (given), tensile cracks will form, and calculation must adapt the cracked transformed section.

Equation 1, $b(kd)^2/2 - nA_s(d - kd) = 0$, with $b = 250 \text{ mm}$, $d = 600 \text{ mm}$, $n = 8$, and $A_s = 1530 \text{ mm}^2$ inserted, gives

$kd = 198 \text{ mm}$ (distance to N.A.)

$k = 198/600 = 0.33$,

$j = 1 - k/3 = 0.89$

$f_s = M / A_s jd = 122,000,000 / [1530 \times 0.89 \times 600] = 149.3 \text{ MPa}$

$f_c = 2M / k j b d^2 = 2 \times 122,000,000 / [0.33 \times 0.89 \times 250 \times 600^2] = 9.23 \text{ MPa}$

or, another way to find k :

$\rho = A_s / bd = 1530 / (250 \times 600) = 0.0102$, $\rho n = 0.0102 \times 8 = 0.0816$

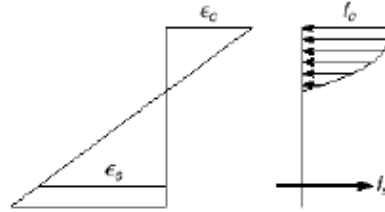
$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = \sqrt{(0.0816)^2 + 2(0.0816)} - 0.0816 = 0.33$ as before.

Notes: (compared with Example 1; doubling M)

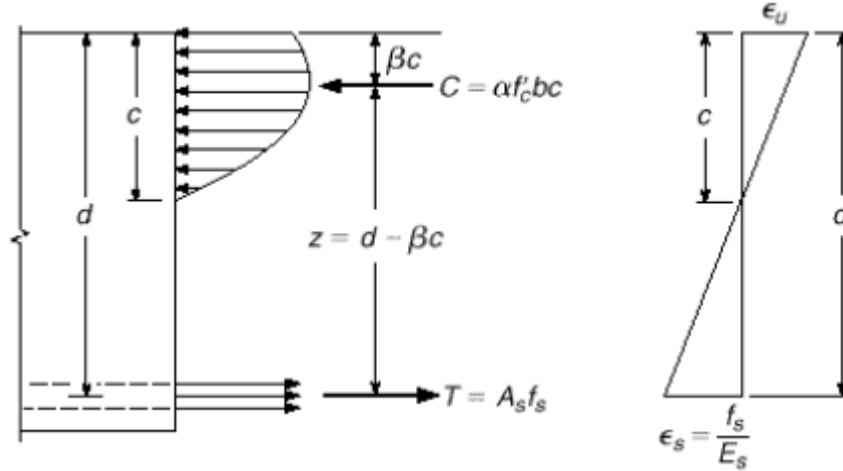
1. N.A. has moved upward: changed from 342 to 198 mm.
2. The steel stress changed from 19.43 to 149.3 MPa (about 8 times).
3. The concrete compressive stress has increased from 3.22 to 9.23 MPa (about 3 times).
4. The moment of inertia of cracked section ($2,625,000,000 \text{ mm}^4$ check this!) is less than that of uncracked section ($6,481,000,000 \text{ mm}^4$). This affects the magnitude of the deflection.

c) Flexural Strength (figure f)

At high loads, close to failure, the distribution of stresses and strains is that of fig. f:



Stress and strain distributions at ultimate load are assumed as shown in fig. below:



For failure mode A, two criteria are implied

- $f_s = f_y$

- The concrete crushes when the maximum strain reaches $\epsilon_u = 0.003$.

It is necessary to know, for a given distance c of N.A.,

1. The total resultant compression force C in the concrete.
2. Its vertical location, i.e., its distance from the outer compression fiber.

In rectangular beams, area in compression is bc , and $C = f_{av}bc$

Let $\alpha = f_{av} / f'_c$ then $C = \alpha f'_c bc$

The location of C is at βc from top.

Knowing α and β will define the compressive stresses.

If α and β are known, then equilibrium requires that

$$C = T \quad \text{or} \quad \alpha f'_c bc = A_s f_s$$

Also

$$M = Tz = A_s f_s (d - \beta c)$$

$$\text{Or } M = \alpha f'_c bc(d - \beta c)$$

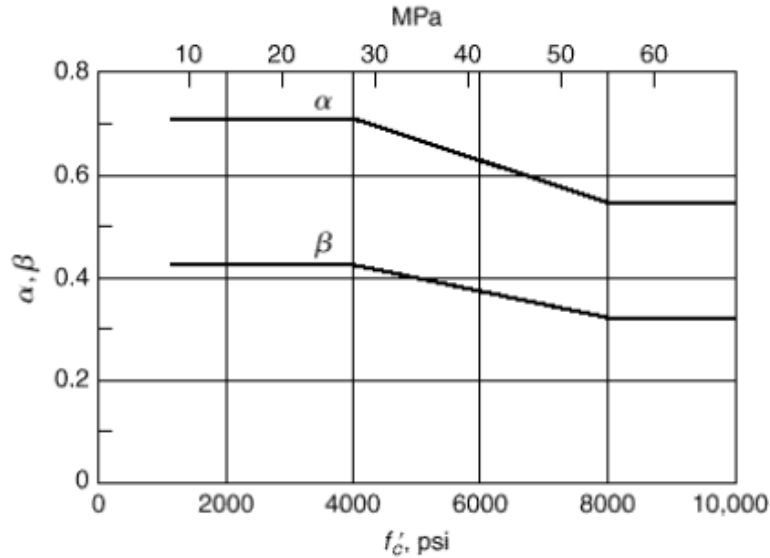
Set $f_s = f_y$ then $c = A_s f_y / \alpha f'_c b$

Using $A_s = \rho bd$, then $c = \rho f_y d / \alpha f'_c$

Substitute, M_n is then obtained

$$M_n = \rho f_y b d^2 (1 - \beta \rho f_y / \alpha f'_c)$$

From extensive experimental work, the values of α and β have shown to be as in the figure below (for $f'_c \leq 28$ MPa, $\alpha = 0.72$ and $\beta = 0.425$)



Now, the nominal moment equation becomes:

$$M_n = \rho f_y b d^2 (1 - 0.59 \rho f_y / f'_c)$$

Balanced reinforcement ratio ρ_b

The balanced reinforcement ratio, ρ_b represents that amount of reinforcement necessary for the beam to fail by crushing of the concrete at the same load that causes the steel to yield.

Hooke's law: $f_s = \epsilon_s E_s$

From strain distribution (see fig.), similar triangles give

$$f_s = \epsilon_u E_s (d - c) / c$$

Setting $f_s = f_y$, and substituting ϵ_y for f_y / E_s , the value of c defining the unique position of the N.A. corresponding to simultaneous crushing of the concrete and initiation of yielding in the steel,

$$c = d \cdot \epsilon_u / (\epsilon_u + \epsilon_y)$$

Substituting c in equation $C = T$ or $\alpha f'_c b c = A_s f_s$ with $A_s f_s = \rho b d f_y$, the ρ_b is obtained

$$\rho_b = (\alpha f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y)]$$

Example 3

Determine the nominal moment M_n at which the beam of Examples 1 and 2 will fail.

Solution

$$\rho = A_s / bd = 1530 / (250 \times 600) = 0.0102 \text{ (always write } \rho \text{ with 4 digits)}$$

check

$$\rho_b = (\alpha f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y)] = 0.0282 \quad (\alpha = 0.72)$$

Since $\rho < \rho_b$, the beam will fail in tension by yielding of the steel, its nominal moment is

$$\begin{aligned} M_n &= \rho f_y b d^2 (1 - 0.59 \rho f_y / f'_c) \\ &= 0.0102 \times 420 \times 250 \times 600^2 (1 - 0.59 \times 0.0102 \times 420 / 28) \\ &= 350,800,000 \text{ N-mm} = 350.8 \text{ kN-m} \end{aligned}$$

At this M_n , the distance to neutral axis is

$$\begin{aligned} c &= \rho f_y d / \alpha f'_c \\ &= 0.0102 \times 420 \times 600 / (0.72 \times 28) = 127.5 \text{ mm} \end{aligned}$$

Summary

	Ex 1:Uncracked	Ex 2:Cracked	Ex 3:Ultimate
NA from top,mm	342	198	127.5
f_c / f_s (MPa/MPa)	3.22 / 19.43	9.23 / 149.3	28 / 420
M , kN-m	61	122	350.8

The differences between various stages (as the load is increased) are

1. The migration of the N.A. toward the compression edge.
2. The increase in steel stress.
3. The increase in concrete compressive stress.

Design of Tension-Reinforced Rectangular Beams

To provide sufficient strength to RC structures:

1. The nominal strength is modified by a **strength reduction factor** ϕ , less than unity, to obtain the *design strength*.
2. The *required strength* is found by applying **load factors** γ , greater than unity, to loads actually expected (*service loads*).

Thus, RC members are proportioned such that $M_u \leq \phi M_n$; $V_u \leq \phi V_n$; $P_u \leq \phi P_n$ where subscripts n denotes the nominal strengths in flexure, shear, and axial load respectively, and u denote the factored load moment, shear and axial load.

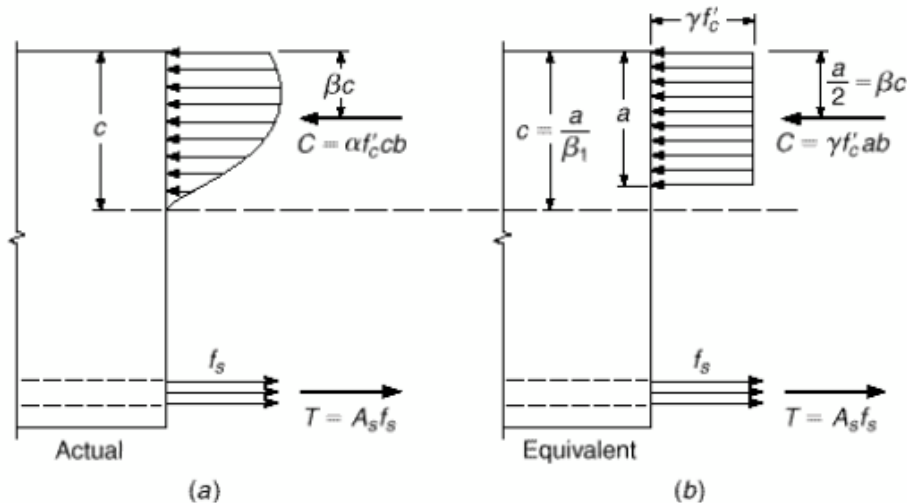
See page 4, chapter 1 of the lecture notes.

a. Equivalent Rectangular Stress Distribution

It was noted that the actual shape of the concrete compressive stress distribution varies considerably. The magnitude C and location βc of the resultant of the concrete compressive stresses are obtained from experiments and expressed in the two parameters α and β .

For simplicity, the actual stress distribution is replaced by an equivalent one of simple rectangular outline. See next figure.

The conditions are that the magnitude of C and its location must be the same in the equivalent rectangular as in the actual stress distribution.



$$C = \alpha f'_c b c = \gamma f'_c a b \quad \text{from which} \quad \gamma = \alpha c / a$$

With $a = \beta_1 c$, this gives $\gamma = \alpha / \beta_1$

The force C is located at the same distance: $\beta_1 = 2\beta$.

$\gamma = \alpha / \beta_1 = \alpha / 2\beta$ is seen independent of f'_c and can be taken as 0.85 throughout (e.g. $0.72/(2 \times 0.425) = 0.85$):

The force C : $C = 0.85 f'_c a b$

The distance a : $a = \beta_1 c$

$\beta_1 = 0.85$ for $f'_c \leq 28 \text{ MPa}$

$\beta_1 = 0.85 - 0.05 (f'_c - 28) / 7$ for $f'_c > 28 \text{ MPa}$

$0.65 \leq \beta_1 \leq 0.85$

b. Balanced Strain Condition

From strain diagram $c = d \cdot \epsilon_u / (\epsilon_u + \epsilon_y)$

Equilibrium $C = T$; $0.85 \beta_1 f'_c b c = \rho b d f_y$

$$\rho_b = 0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y)]$$

c. Under-reinforced Beams

To ensure that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete, this can be done, theoretically by requiring $\rho < \rho_b$

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons:

1. To get significant yielding before failure.
2. Material properties are never known exactly.
3. Strain-hardening of the steel may lead to concrete compression failure.
4. Actual steel area provided will always be equal to or larger than required.
5. Lower ρ increases deflection and thus provides warning prior to failure.

d. ACI Code Provisions for Under-reinforced Beams

ACI Code defines the safe limit of maximum reinforcement based on the **net tensile strain** ϵ_t of the reinforcement farthest from the compression face of the concrete at depth d_t

1. The minimum tensile reinforcement strain allowed at nominal strength:

$$\epsilon_t = \epsilon_u (d_t - c) / c$$

$$\rho = 0.85 \beta_1 (f'_c / f_y) (d_t / d) [\epsilon_u / (\epsilon_u + \epsilon_t)]$$

conservatively

$$\rho = 0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_t)]$$

To ensure under-reinforced behavior, ACI Code 21.2 establishes a minimum net tensile strain ϵ_t at nominal of $(\epsilon_y + 0.003)$ for members subjected to axial loads less than $0.10 f'_c A_g$, where A_g is the gross area of the cross section. Substituting in ρ equation:

$$\rho_{max} = 0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y + 0.003)]$$

2. Allowing **strength reduction factors** that depend on the tensile strain at nominal strength. The Code defines:
 - a. **Tension-controlled member:** The one with a net tensile strain $\epsilon_t \geq (\epsilon_y + 0.003)$. the corresponding strength reduction factor $\phi = 0.9$.
 - b. **Compression-controlled member:** The one with a net tensile strain $\epsilon_t \leq 0.002$. The corresponding strength reduction factor $\phi = 0.65$. For spirally-reinforced members $\phi = 0.75$

For ϵ_t between ϵ_y and $(\epsilon_y + 0.003)$, ϕ varies linearly, and ACI Code allows linear interpolation of ϕ based on ϵ_t . see Table below:

Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

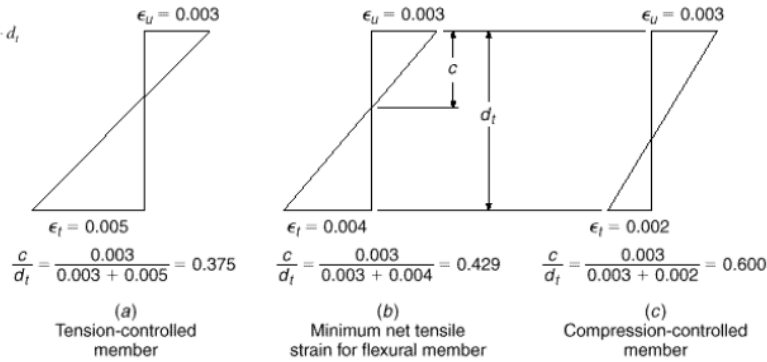
Net tensile strain ϵ_t	Classification	ϕ			
		Type of transverse reinforcement			
		Spirals conforming to 25.7.3		Other	
$\epsilon_t \leq \epsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\epsilon_{ty} < \epsilon_t < \epsilon_{ty} + 0.003$	Transition ^[1]	$0.75 + 0.15 \frac{(\epsilon_t - \epsilon_{ty})}{(0.003)}$	(c)	$0.65 + 0.25 \frac{(\epsilon_t - \epsilon_{ty})}{(0.003)}$	(d)
$\epsilon_t \geq \epsilon_{ty} + 0.003$	Tension-controlled	0.90	(e)	0.90	(f)

^[1]For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

The depth of equivalent rectangular stress block a :

Since $c = a / \beta_1$, it is more convenient to compute c/d_t rather than ρ or net ϵ_t , see

Figure. Maximum value of $c/d_t = 0.375$ for $\epsilon_t \geq 0.005$

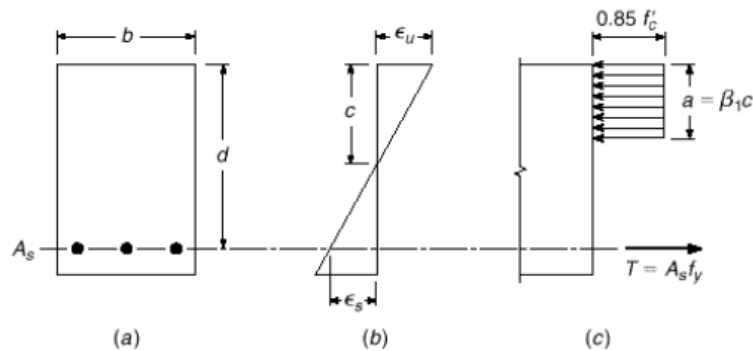
FIGURE 3.10Net tensile strain and c/d_t ratios.

The nominal flexural strength is given by (see figure below)

$$M_n = A_s f_y (d - a/2), \quad a = A_s f_y / 0.85 f'_c b$$

FIGURE 3.11

Singly reinforced rectangular beam.



Example 4

Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 3. Recall $b = 250$ mm, $d = 600$ mm, $A_s = 1530$ mm², $f'_c = 28$ MPa, $f_y = 420$ MPa.

Solution

$$\beta_1 = 0.85 \quad (f'_c = 28 \text{ MPa})$$

$$\rho_{\max} = 0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y + 0.003)]$$

$$= 0.85 \times 0.85 (28/420) [0.003 / (0.003 + 0.005)] = 0.0181$$

$$\text{Actual } \rho = 1530 / (250 \times 600) = 0.0102$$

Since $\rho < \rho_{\max}$, the member will fail by yielding of steel.

Alternatively, recall $c = 127.5$ mm,

$c/d_t = 127.5/600 = 0.213 < 0.375$, the member will fail by yielding of steel

$$a = A_s f_y / 0.85 f'_c b = 1530 \times 420 / (0.85 \times 28 \times 250) = 108 \text{ mm}$$

$$M_n = A_s f_y (d - a/2) = 1530 \times 420 (600 - 108/2) = 350.9 \times 10^6 \text{ Nmm} = \underline{350.9 \text{ kNm}}$$

Moment equation can be re written (as derived previously) as follows:

$$M_n = \rho f_y b d^2 (1 - 0.59 \rho f_y / f'_c)$$

$$= 0.0102 \times 420 \times 250 \times 600^2 (1 - 0.59 \times 0.0102 \times 420 / 28) [10^{-6}] = \underline{350.8 \text{ kNm}}$$

This equation: $M_n = \rho f_y b d^2 (1 - 0.59 \rho f_y / f'_c)$ may be simplified further for everyday design as follows

$$M_n = R b d^2$$

In which

$$R = \rho f_y (1 - 0.59 \rho f_y / f'_c) \quad (\text{MPa})$$

The values of the *flexural resistance factor* R are tabulated in Appendix A5 (Nilson)

In accordance with safety the safety provisions of the ACI Code, the *nominal flexural strength* M_n is reduced by imposing the strength reduction factor ϕ to obtain the *design strength* ϕM_n

$$\phi M_n = \phi A_s f_y (d - a/2)$$

Or, alternatively, $\phi M_n = \phi \rho f_y b d^2 (1 - 0.59 \rho f_y / f'_c)$

Or $\phi M_n = \phi R b d^2$

Example 4 (continued): Since $\rho < \rho_{\max}$ (or $c/d_t < 0.375$), then $\varepsilon_t > (\varepsilon_y + 0.003)$. Therefore, $\phi = 0.9$ and design capacity is $\phi M_n = 0.9 \times 350.9 = 315.8 \text{ kNm}$

e. Minimum Reinforcement Ratio

In very lightly reinforced beams, if the flexural strength < the moment that produce cracking, the beam will fail immediately and without warning upon formation of the first flexural crack.

To ensure against this type of failure, a **lower limit** can be established for the reinforcement.

According to ACI Code 9.6, at any section where tensile reinforcement is required by analysis, the area A_s provided must not be less than

$$A_{s,min} = \rho_{min} b_w d$$

$$\rho_{min} = 0.25 \sqrt{f'_c} / f_y \geq 1.4 / f_y$$

$$f_y \leq 560 \text{ MPa}$$

f. Examples of Rectangular Beams

Example 5 (Analysis problem)

A rectangular beam has width **300 mm** and effective depth **490 mm**. it is reinforced with **4 No.29 (#9)** bars in one row. If $f_y = 420 \text{ MPa}$ and $f'_c = 28 \text{ MPa}$, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to ACI Code?

Solution:

Area of 4 No.29 bars = $4 \times 645 = 2580 \text{ mm}^2$ (Table A2, Nilson)

$a = A_s f_y / 0.85 f'_c b = 2580 \times 420 / (0.85 \times 28 \times 300) = 151.8 \text{ mm}$

$c = a / \beta_1 = 151.8 / 0.85 = 178.5 \text{ mm}$

$$c / d_t = 178.5 / 490 = 0.364 (< 0.375), \text{ i.e. } (\epsilon_t < \epsilon_y + \mathbf{0.003})$$

Thus, the beam is under-reinforced

$$\text{or, } \rho = A_s / bd = 2580 / (300 \times 490) = \mathbf{0.0176}$$

$$\rho_{\max} = \mathbf{0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y + \mathbf{0.003})]}$$

$$= 0.85 \times 0.85 (28/420) [0.003 / (0.003 + 0.002 + 0.003)] = 0.0181$$

Since $c / d_t = 0.364 (< 0.375)$ and $\rho = A_s / bd = 0.0176 < \rho_{\max} = 0.0181$

Then $\phi = \mathbf{0.90}$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.90 \times 2580 \times 420 (490 - 151.8/2)$$

$$= 0.90 \times 448.7 \times 10^6 \text{ Nmm} = \mathbf{403.8 \text{ kNm}}$$

Check $\rho_{\min} = 0.25 \sqrt{28} / 420 \geq 1.4 / 420 = 0.0033$. Thus $\rho_{\min} < \rho = 0.0176 < \rho_{\max}$ is satisfactory.

Example 6 (Design problem)

Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of **4.5 m** that is to carry a computed dead load of **20 kN/m** and a service live load of **31 kN/m**. Material strengths are $f'_c = \mathbf{28 \text{ MPa}}$ and $f_y = \mathbf{420 \text{ MPa}}$.

Solution:

Factored load $w_u = 1.2 D + 1.6 L$

$$= 1.2 \times 20 + 1.6 \times 31 = 73.6 \text{ kN/m}$$

$$M_u = w_u l^2 / 8 = 73.6 \times 4.5^2 / 8 = 186.3 \text{ kNm}$$

To minimize section dimensions, it is desirable to select the maximum permissible reinforcement ratio:

$$\rho_{\max} = \mathbf{0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y + \mathbf{0.003})]}$$

$$= 0.85 \times 0.85 (28/420) [0.003 / (0.003 + \epsilon_y + 0.003)] = 0.0181$$

$$\phi = 0.9 (\epsilon_t = \epsilon_y + \mathbf{0.003})$$

$$M_u = \phi M_n$$

$$186.3 \times 10^6 = 0.9 \times 0.0181 \times 420 b d^2 (1 - 0.59 \times 0.0181 \times 420 / 28)$$

$$b d^2 = 32,420,000 \text{ mm}^2$$

Say $b = 250 \text{ mm}$, $d = 360 \text{ mm}$, then

$$A_s, \text{ required} = 0.0181 \times 250 \times 360 = 1630 \text{ mm}^2$$

USE 2 No.32 (1638 mm²)

Total depth of section $h = d_t + d_b/2 + d_b (\text{stirrup}) + \text{concrete cover}$

$$= 360 + 16 + 13 + 40 = 429 \text{ mm}$$

Round – up to the nearest 25 mm: $h = \mathbf{450 \text{ mm}}$.

Note:

1. The effective depth will be increased: $d = 450 - 40 - 13 - 16 = 381 \text{ mm}$.
Improved economy may be possible by refining the steel area based on the actual, larger d .
2. Infinite number of solutions is possible depending upon the reinforcement ratio selected.

Example 7: (Design problem: section dimensions are given, A_s is required)

Find the steel area required to resist a moment M_u of **150 kNm** using a concrete section having $b = \mathbf{250 \text{ mm}}$, $d = \mathbf{435 \text{ mm}}$, and $h = \mathbf{500 \text{ mm}}$.

$f'_c = \mathbf{28 \text{ MPa}}$ and $f_y = \mathbf{420 \text{ MPa}}$.

Solution:

Assume $a = 100$ mm

$$\phi M_n = \phi A_s f_y (d - a/2)$$

$$A_s = \phi M_n / \phi f_y (d - a/2) = 150 \times 10^6 / 0.9 \times 420 (435 - 100/2) = 1031 \text{ mm}^2$$

$$\text{Check } a = A_s f_y / 0.85 f'_c b = 1031 \times 420 / 0.85 \times 28 \times 250 = 72.8 \text{ mm}$$

Next assume $a = 70$ mm and recalculate A_s :

$$A_s = 150 \times 10^6 / 0.9 \times 420 (435 - 70/2) = 992 \text{ mm}^2$$

No further iteration is required. **USE $A_s = 992 \text{ mm}^2$ (2 No.25 bars $A_s = 1080 \text{ mm}^2$)**

Check $\rho = 0.0091 < \rho_{max}$, then $\phi = 0.9$ OK

Example 8: (section dimensions are given, A_s is required ; effect of ϕ)

A **6 m** long simple span beam of height = **400 mm** and the width = **300 mm**. The following loads and material properties are given: $w_d = 11 \text{ kN/m}$, $w_l = 24 \text{ kN/m}$, $f'_c = 35 \text{ MPa}$, and $f_y = 420 \text{ MPa}$. Determine the reinforcement of the beam.

Solution:

$$\text{Factored load } w_u = 1.2 \times 11 + 1.6 \times 24 = 51.6 \text{ kN/m}$$

$$M_u = w_u l^2 / 8 = 51.6 \times 6^2 / 8 = 232.2 \text{ kNm}$$

Assume $a = 100$ and $\phi = 0.9$

$d = 400 - 65 = 335$ mm (assuming 65 mm concrete cover from centroid of bars)

$$A_s = M_u / \phi f_y (d - a/2) = 232.2 \times 10^6 / [0.9 \times 420 (335 - 50)] = 2156 \text{ mm}^2$$

Try 2 No.32 bars and 1 No. 29 bar, $A_{s, \text{provided}} = 2283 \text{ mm}^2$

$$\text{Check } a = A_s f_y / 0.85 f'_c b = 2283 \times 420 / 0.85 \times 35 \times 300 = 107.4 \text{ mm} \quad 107.4 \text{ mm} > 100 \text{ mm assumed; continue}$$

$$M_n = A_s f_y (d - a/2) = 2283 \times 420 (335 - 107.4/2) 10^{-6} = 269.7 \text{ kNm}$$

$$M_u = \phi M_n = 0.9 \times 269.7 = 242.7 \text{ kNm (adequate: } > 232.2 \text{ applied)}$$

To validate the selection of $\phi = 0.9$, the net ϵ_t must be checked:

$$c = a / \beta_1 = 107.4 / 0.80 = 134.3 \text{ mm.}$$

$$c/d = 134.3/335 = 0.401 > 0.375 \text{ so } \epsilon_t > \epsilon_y + 0.003 \text{ is not satisfied.}$$

Try increasing the height to 450 mm;

Repeating the calculations: say $a = 90$ mm

$$A_s = M_u / \phi f_y (d - a/2) = 232.2 \times 10^6 / [0.9 \times 420 (385 - 45)] = 1807 \text{ mm}^2$$

Try 3 No.29 bars, $A_{s, \text{provided}} = 1935 \text{ mm}^2$

$$a = A_s f_y / 0.85 f'_c b = 1935 \times 420 / 0.85 \times 35 \times 300 = 91 \text{ mm ok}$$

$$c = a / \beta_1 = 91 / 0.80 = 114 \text{ mm}$$

$$c/d = 114/385 = 0.296 < 0.375 \text{ so } \epsilon_t < \epsilon_y + 0.003 \text{ (satisfied).}$$

Thus, $\phi = 0.90$

$$M_n = A_s f_y (d - a/2) = 1935 \times 420 (385 - 91/2) 10^{-6} = 275.9 \text{ kNm}$$

$$M_u = \phi M_n = 0.90 \times 275.9 = 248.3 \text{ kNm (adequate } > 232.2)$$

g. Over-reinforced Beams

Occasionally, it may be necessary to calculate the flexural strength of an over-reinforced compression controlled member for which $f_s < f_y$ at flexural failure.

$$\phi M_n = \phi A_s f_s (d - a/2) \quad \phi = 0.65 \quad \& \quad f_s = \epsilon_s E_s$$

In this case, $\epsilon_s < \epsilon_y$; in terms of ϵ_u and c :

$$\text{From strain diagram; } \epsilon_s = \epsilon_u (d - c) / c \quad \dots (1)$$

$$\text{From equilibrium; } 0.85 \beta_1 f'_c b c = \epsilon_s E_s b d \quad \dots (2)$$

Solve simultaneously to find ϵ_s

h. Design Aids

In practice, the design of beams and other RC members is greatly facilitated by the use of *design aids*.

Tables A.1, A.2, A.4 through A.7, and Graph A.1 relate directly to this chapter.

For design purposes, there are two possible approaches:

1st Approach: Start with selecting the optimum ρ and then calculating concrete dimensions, as follows:

1. Set $M_u = \phi R b d^2$
2. **Table A.4:** Select an appropriate ρ between ρ_{max} and ρ_{min} . Often a ratio of about (0.6 ρ_{max}) will be an economical and practical choice.
If $\rho \leq \rho_{max}$ then $\phi = 0.9$
If $\rho > \rho_{max}$ then increase h and recalculate, or use compression reinf. (p.18)
3. **Table A.5:** Find the flexural resistance factor R . Then $b d^2 = M_u / \phi R$
4. Choose b and d to meet that requirement. Often $d = 2$ to 3 times b is appropriate.
5. Calculate $A_s = \rho b d$, then use **Table A.2** to choose the size and no. of bars.
6. Refer to **Table A.7** to ensure that the selected beam width will provide room for the bars chosen, with adequate concrete cover and spacing.

2nd Approach: Start with selecting concrete dimensions, after which the required reinforcement is found, as follows:

1. Select b and d , then calculate $R = M_u / \phi b d^2$
2. Use **Table A.5** to find $\rho < \rho_{max}$
3. Calculate $A_s = \rho b d$ then use **Table A.2** to choose the size and no. of bars.
4. Refer to **Table A.7** to ensure that the selected beam width will provide room for the bars chosen, with adequate concrete cover and spacing.

Example 9: USE DESIGN AIDS

A rectangular beam has width **300 mm** and effective depth **490 mm**. It is reinforced with **4 No.29 (#9)** bars in one row. If $f_y = 420 \text{ MPa}$ and $f'_c = 28 \text{ MPa}$, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to ACI Code?

Solution:

From Table A.2, 4 No.29 bars provide $A_s = 2580 \text{ mm}^2$

$$\rho = A_s / b d = 0.0176$$

Table A.4: this ρ is below ρ_{max} (0.0181) and above ρ_{min} (0.0033)

Table A.5b: $R = 6.24 \text{ MPa}$ (by interpolation bet. 6.07 & 6.36)

$$M_u = \phi M_n = 0.90 R b d^2 = 404.5 \text{ kNm}$$

Example 10: USE DESIGN AIDS

Find the concrete cross section and the steel area required for a simply supported rectangular beam. $M_u = 186.3$ kNm. Material strengths are $f'_c = 28$ MPa and $f_y = 420$ MPa.

Solution:

Table A.4, $\rho_{max} = 0.0181$. For economy $\rho = 0.6(0.0181) = 0.0109$.

Table A.5a, by interpolation $R = 4.14$ MPa. Then

$$bd^2 = M_u / \phi R = 50 \times 10^6 \text{ mm}^3$$

$b = 250$ mm and $d = 447.2$ mm will satisfy this, but the depth will be rounded to 450 mm, to provide a total depth of 520 mm. It follows that

$$R = M_u / \phi b d^2 = 4.09 \text{ MPa}$$

Table A.5a, by interpolation, $\rho = 0.0108$

$$A_s = 0.0108 (250)(450) = 1215 \text{ mm}^2 \text{ (Use 2 No.29; } A_s = 1290 \text{ mm}^2)$$

Example 11: USE DESIGN AIDS

Find the steel area required to resist a moment M_u of 150 kNm using a concrete section having $b = 250$ mm, $d = 435$ mm, and $h = 500$ mm.

$f'_c = 28$ MPa and $f_y = 420$ MPa.

Solution:

$$R = M_u / \phi b d^2 = 3.52 \text{ MPa}$$

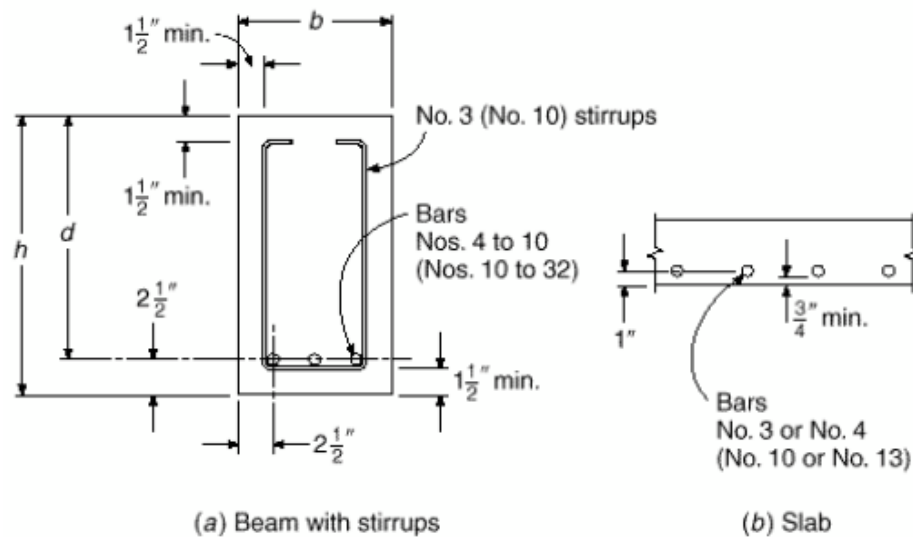
Table A.5a, $\rho = 0.0091$ giving $A_s = 0.0091(250)(435) = 990 \text{ mm}^2$

USE 2 No.25 bars. ($A_s = 1020 \text{ mm}^2$)

Practical Considerations in the Design of Beams**a. Concrete Protection for Reinforcement**

To provide the steel with adequate concrete protection against fire and corrosion, the designer must maintain a certain minimum thickness of concrete cover outside of the outermost steel. The thickness required will vary, depending upon the type of member and conditions of exposure.

The requirements of concrete cover in beams and slabs are shown in figure below:



$$\frac{3}{4} \text{ in} = 20 \text{ mm}, 1 \text{ in} = 25 \text{ mm}, 1\frac{1}{2} \text{ in} = 40 \text{ mm}, 2\frac{1}{2} \text{ in} = 65 \text{ mm}$$

b. Selection of Bars and Bar Spacing

It is often desirable to mix bar sizes to meet steel area requirements more closely. In general, mixed bars should be of comparable diameter, and generally should be arranged symmetrically about the vertical centerline.

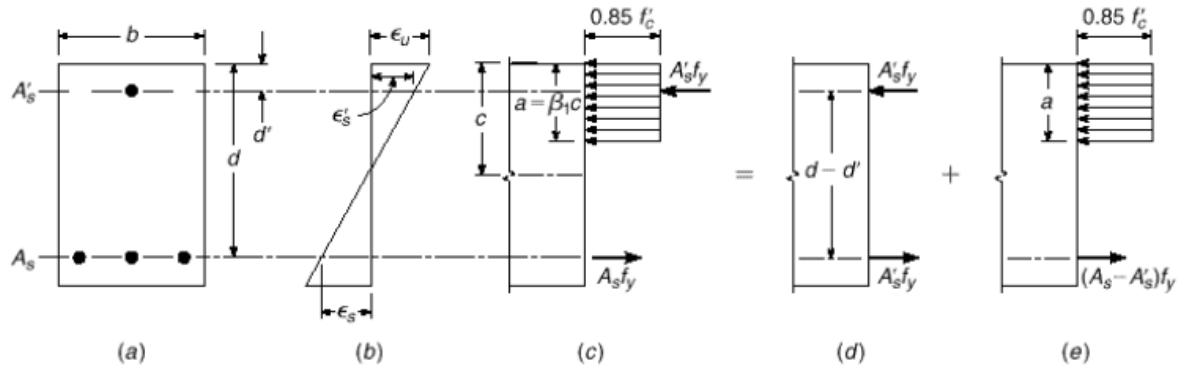
ACI Code 25.2 specifies that the minimum clear distance between adjacent bars shall not be less than the nominal diameter of the bars d_b , **25 mm**, or $(4/3 d_{agg.})$. (for columns $1\frac{1}{2} d_b$ or **40 mm**).

Where beam reinforcement is placed in two or more layers, the clear distance between layers must not be less than 25 mm, and the bars in the upper layer should be placed directly above those in the bottom layer.

The maximum number of bars that can be placed in a beam of given width is limited by bar diameter, by concrete cover, and by the maximum size of aggregate specified. See **Table A.7**.

Rectangular Beams with Tension and Compression Reinforcement

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete cannot develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a **doubly reinforced** beam, i.e., with compression as well as tension reinforcement (see Fig.):



a. Tension and Compression Steel Both at Yield Stress

In a doubly reinforced beam;

If $\rho \leq \rho_{max}$, disregard the compression bars.

If $\rho > \rho_{max}$, the total resisting moment is the sum of two parts:

The 1st part: $M_{n1} = A'_s f_y (d - d')$

The 2nd part: $M_{n2} = (A_s - A'_s) f_y (d - a/2)$

$$a = (A_s - A'_s) f_y / 0.85 f'_c b$$

$$\rho = A_s / bd \quad \text{and} \quad \rho' = A'_s / bd$$

$$a = (\rho - \rho') f_y d / 0.85 f'_c$$

The total nominal moment:

$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y (d - a/2)$$

The maximum reinforcement ratio: ($\phi = 0.9$)

$$\rho_{max}^- = \rho_{max} + \rho'$$

Example 12: (Analysis problem)

A rectangular beam has a width of **300 mm** and an effective depth to the centroid of the tension reinforcement of **600 mm**. The tension reinforcement consists of **three No.32** bars and **three No.29, in two rows**. Compression reinforcement consisting of **two No.19** bars is placed **65 mm** from the compression face of the beam. If $f'_c = 35$ MPa and $f_y = 420$ MPa, what is the design moment capacity of the beam?

Solution:

$$A_s = 4392 \text{ mm}^2, \quad \rho = 4392 / (300 \times 600) = 0.0244$$

$$A'_s = 568 \text{ mm}^2, \quad \rho' = 568 / (300 \times 600) = 0.0032$$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded:

$$\rho_{max} = 0.85 \beta_1 (f'_c / f_y) [\epsilon_u / (\epsilon_u + \epsilon_y + 0.003)] = 0.0213 \text{ (or use Table A.4)}$$

Actual $\rho = 0.0244 > \rho_{max}$, so the beam must be analyzed as a doubly reinforced.

$$\begin{aligned} \rho_{cy}^- &= 0.85 \beta_1 (f'_c / f_y) (d' / d) [\epsilon_u / (\epsilon_u - \epsilon_y)] + \rho' \\ &= 0.85 \times 0.8 \times (35/420) \times (65/600) \times [0.003 / (0.003 - 0.0021)] + 0.0032 = 0.0237 \end{aligned}$$

Actual $\rho = 0.0244 > \rho_{cy}^-$, so the compression bars will yield when the beam fails.

$$\rho_{max}^- = \rho_{max} + \rho' = 0.0213 + 0.0032 = 0.0245$$

Actual $\rho = 0.0244 < \rho_{max}^-$, as required. Then

$$\begin{aligned} a &= (A_s - A'_s) f_y / 0.85 f'_c b \\ &= (4392 - 568) 420 / (0.85 \times 35 \times 300) = 180 \text{ mm} \end{aligned}$$

$$c = a / \beta_1 = 180 / 0.80 = 225 \text{ mm}$$

$$\epsilon_t = \epsilon_u (d - c) / c = 0.003 (600 - 225) / 225 = 0.005 = \epsilon_y + 0.003, \text{ thus } \phi = 0.90 \text{ OK}$$

$$\begin{aligned} M_n &= M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y (d - a/2) \\ &= [568 \times 420 (600 - 65) + 3824 \times 420 (600 - 180/2)] \times 10^{-6} = 946.7 \text{ kNm} \end{aligned}$$

Design strength is

$$\phi M_n = 0.90 \times 946.7 = \underline{852 \text{ kNm}}$$

b. Compression Steel below Yield Stress

In many cases, the compression bars will be below the yield stress at failure ($f'_s < f_y$).

From geometry of the strain diagram, fig. b:

$$c = d' [\epsilon_u / (\epsilon_u - \epsilon_y)]$$

Sum of forces in fig. c gives ρ_{cy}^- : the minimum ratio that will ensure yielding of compression steel,

$$\rho_{cy}^- = 0.85 \beta_1 (f'_c / f_y) (d' / d) [\epsilon_u / (\epsilon_u - \epsilon_y)] + \rho'$$

$$\text{From figures b and c: } \rho_{max}^- = \rho_{max} + \rho' (f'_s / f_y)$$

where

$$f'_s = E_s \epsilon'_s = E_s [\epsilon_u - (d' / d) (\epsilon_u + \epsilon_y)] \leq f_y$$

for $\epsilon_t = (\epsilon_y + 0.003)$; $\phi = 0.9$

$$f'_s = E_s \epsilon'_s = E_s [\epsilon_u - (d' / d) (\epsilon_u + \epsilon_y + 0.003)] \leq f_y$$

Hence, the max reinforcement ratio is

$$\rho_{max}^- = \rho_{max} + \rho' (f'_s / f_y)$$

If the tensile reinforcement ratio is less than ρ_{max}^- and less than ρ_{cy}^- , then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed:

From strain diagram: $f'_s = \epsilon_u E_s (c - d') / c$

From Equilibrium: $A_s f_y = 0.85 \beta_1 f'_c b c + A'_s \epsilon_u E_s (c - d') / c$

Solve for c , and knowing $a = \beta_1 c$

$$M_n = 0.85 f'_c a b (d - a/2) + A'_s f'_s (d - d')$$

Example 13: (Design problem)

A rectangular beam that must carry a service live load of **36 kN/m** and a calculated dead load of **15.3 kN/m** on an **5.5 m** simple span is limited in cross section for architectural reasons to **250 mm** width and **500 mm** (20 in) total depth. If $f'_c = 28$ MPa and $f_y = 420$ MPa, what steel area(s) must be provided?

Solution:

Factored load, $w_u = 1.2 \times 15.3 + 1.6 \times 36 = 75.96$ kN/m

$$M_u = 75.96 (5.5)^2 / 8 = 287.2 \text{ kNm}$$

Assume tension steel centroid is 100mm above the bottom face and assume compression steel, if required, be placed 65mm below the top surface. Then $d = 400\text{mm}$, $d' = 65\text{mm}$

Check if the section is singly reinforced:

Table A.4: $\rho_{max} = 0.0181$ for $\phi = 0.9$

$$A_s = 250 \times 400 \times 0.0181 = 1810 \text{ mm}^2$$

$$a = 1810 \times 420 / (0.85 \times 28 \times 250) = 127.8 \text{ mm}$$

$$c = a / \beta_1 = 127.8 / 0.85 = 150.3 \text{ mm}$$

$$M_n = A_s f_s (d - a/2) = 1810 \times 420 (400 - 127.8/2) 10^{-6} = 255.5 \text{ kNm}$$

(Alternatively, Table A.5b, $R = 6.39$, $M_n = R b d^2 = 255.5 \text{ kNm}$)

$\phi M_n = 230 \text{ kNm} < 287.2 \text{ kNm}$, therefore compression steel is needed as well as additional tension steel.

The remaining moment to be carried by compression steel couple:

$$M_I = 287.2 - 230 = 57.2 \text{ kNm}$$

From strain diagram

$$\epsilon'_s = 0.003 (150.3 - 65) / 150.3 = 0.00170 < \epsilon_y = 0.002 \text{ [thus } f'_s < f_y]$$

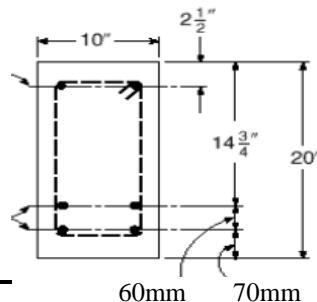
$$f'_s = \epsilon'_s E_s = 0.00170 \times 200000 = 340 \text{ MPa}$$

$$A'_s = 57.2 \times 10^6 / (0.9 [340 (400 - 65)]) = 558 \text{ mm}^2$$

Total area of tensile reinforcement at 420 MPa:

$$A_s = 1810 + 558 (340/420) = 2262 \text{ mm}^2$$

USE **2 No.19 bars** (568 mm²) as compression reinforcement and **4 No.29 bars** (2580 mm²) as tension reinforcement. To place tension reinforcement, 2 rows of 2 bars each are used. See figure. [$2\frac{1}{2}$ in = 65 mm, $14\frac{3}{4}$ in = 370 mm, 10 in = 250 mm, 20 in = 500mm.]



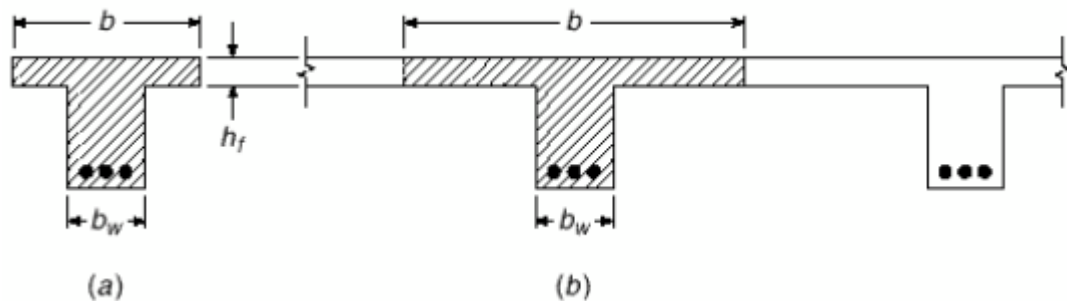
T Beams

RC floors, roofs, decks, etc., are almost always monolithic. Beam stirrups and bent bars extend up into the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression.

The resulting section is T-shaped rather than rectangular.

The slab forms the beam *flange*, while the part of the beam projecting below the slab is called the *web* or *stem*.

a. Effective Flange Width



The effective flange width, b , has been found to depend on the *beam span* l , the *thickness of the slab* h_f , and the *clear distance to the next beam* l_c .

ACI Code requires that:

1. For symmetric T beams:

$$b \leq l/4$$

$$(b - b_w)/2 \leq 8 h_f$$

$$(b - b_w)/2 \leq l_c/2$$

2. For beams having a slab on one side only:

$$(b - b_w) \leq l/12$$

$$(b - b_w) \leq 6 h_f$$

$$(b - b_w) \leq l_c/2$$

3. For isolated beams:

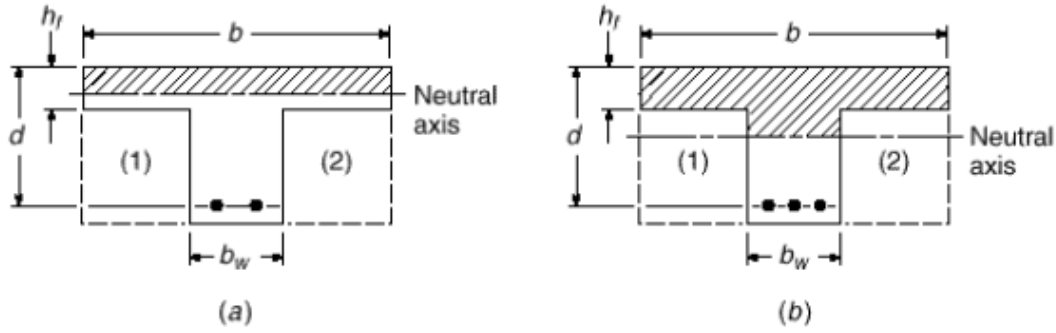
$$h_f \geq b_w/2$$

$$b \leq 4 b_w$$

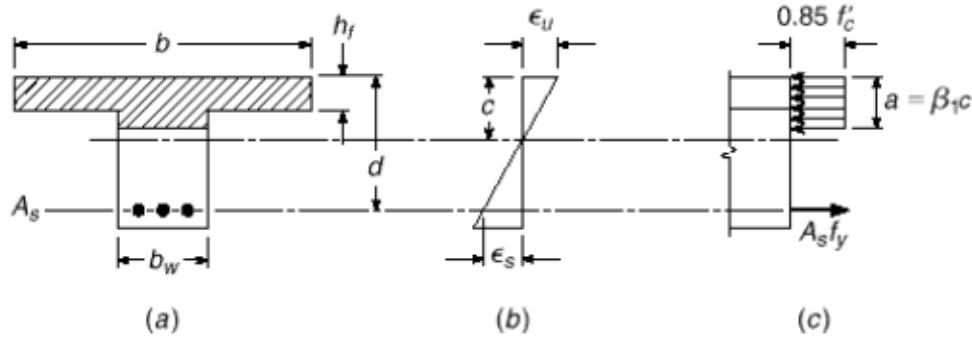
b. Structural Analysis

The neutral axis NA of a T beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials.

If the depth of NA $\leq h_f$, the beam can be analyzed as if it were a rectangular beam of width b . (fig. a). This is because areas (1) and (2) are entirely in tension zone and thus disregarded in flexural calculations.



When the NA is in the web (fig. b), method is developed to account for the actual T-shaped compressive zone.



It is convenient to divide the total tensile steel into two parts. The first part, A_{sf} , represents the steel area required to balance the compressive force in the overhanging portions.

$$A_{sf} = 0.85 f'_c (b - b_w) h_f / f_y$$

$$M_{n1} = A_{sf} f_y (d - h_f / 2)$$

The remaining steel area, $(A_s - A_{sf})$, is balanced by the compression in the rectangular portion of the beam. In this zone:

$$a = (A_s - A_{sf}) f_y / (0.85 f'_c b_w)$$

$$M_{n2} = (A_s - A_{sf}) f_y (d - a / 2)$$

The total nominal resisting moment is the sum of the parts:

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y (d - h_f / 2) + (A_s - A_{sf}) f_y (d - a / 2)$$

From strain diagram:

$$c/d_t \leq \epsilon_u / (\epsilon_u + \epsilon_t)$$

Setting $\epsilon_u = 0.003$ and $\epsilon_t = \epsilon_y + 0.003$ provides a max $c/d_t = 0.375$

The condition of tensile steel yield to occur prior to concrete crushing is satisfied if

$$\rho_{w,max} = \rho_{max} + \rho_f$$

where

$$\rho_w = A_s / b_w d \quad \text{and} \quad \rho_f = A_{sf} / b_w d$$

The minimum reinforcement is based on b_w :

$$\rho_{min} = 0.25 \sqrt{f'_c} / f_y \geq 1.4 / f_y$$

$$A_{s,min} = \rho_{min} b_w d$$

Example 14: (*T Beam Analysis problem*)

An isolated T beam is composed of a flange **700 mm** wide and **150 mm** deep cast monolithically with a web of **250 mm** width that extends **600 mm** below the bottom

surface of the flange to produce a beam of **750 mm** total depth. Tensile reinforcement consists of **6 No.32** bars placed in two horizontal rows. The centroid of the bar group is **650 mm** from the top of the beam. If $f'_c = 21 \text{ MPa}$ and $f_y = 420 \text{ MPa}$, what is the design moment capacity of the beam?

Solution:

Check flange dimensions, $h_f \geq b_w/2 : 150 > 250/2, 150 > 125$

$b \leq 4 b_w : 700 < 4 \times 250, 700 < 1000 \quad \text{OK}$

Area of 6No.32 = 4914 mm^2

Check NA location:

$a = A_s f_y / (0.85 f'_c b) = 4914 \times 420 / (0.85 \times 21 \times 700) = 165.2 \text{ mm} > h_f = 150 \text{ mm}$

so T beam analysis is required.

$A_{sf} = 0.85 f'_c (b - b_w) h_f / f_y = 0.85 \times 21 (700 - 250) \times 150 / 420 = 2869 \text{ mm}^2$

$M_{n1} = A_{sf} f_y (d - h_f/2) = 2869 \times 420 (650 - 150/2) \times 10^{-6} = 692.8 \text{ kNm}$

$a = (A_s - A_{sf}) f_y / (0.85 f'_c b_w) = 2045 \times 420 / (0.85 \times 21 \times 250) = 192.5 \text{ mm}$

$M_{n2} = (A_s - A_{sf}) f_y (d - a/2) = 2045 \times 420 (650 - 192.5/2) \times 10^{-6} = 475.6 \text{ kNm}$

$c = a / \beta_1 = 192.5 / 0.85 = 226.5 \text{ mm}$

$d_t = 685 \text{ mm}$ to the lowest bar,

$c / d_t = 226.5 / 685 = 0.331 < 0.375$, so the $\epsilon_t > (\epsilon_y + 0.003) = 0.005 \rightarrow \phi = 0.9$

$\phi M_n = 0.9 (692.8 + 475.6) = 1052 \text{ kNm}$

Example 15: (T Beam Design problem)

A floor system consists of a **75 mm** concrete slab supported by continuous T beams with a **7.5 m** span, **1.2 m** on centers. Web dimensions are $b_w = 275 \text{ mm}$ and $d = 500 \text{ mm}$.

What tensile steel area is required at midspan to resist a factored moment of **725 kNm** if $f_y = 420 \text{ MPa}$ and $f'_c = 21 \text{ MPa}$?

Solution:

Determine effective flange width:

$b = 16 h_f + b_w = 16 \times 75 + 275 = 1475 \text{ mm} = 1.475 \text{ m}$

$b = l / 4 = 7.5 / 4 = 1.875 \text{ m}$

$b = 1.2 \text{ m}$ (c. / c. spacing)

The controlling $b = 1.2 \text{ m}$

Assume $a = h_f = 75 \text{ mm}$

$d - a/2 = 500 - 75/2 = 462.5 \text{ mm}$

Trial: $A_s = M_u / \phi f_y (d - a/2) = 725 \times 106 / [0.9 \times 420 \times 462.5] = 4147 \text{ mm}^2$

Check $a = A_s f_y / (0.85 f'_c b) = 4147 \times 420 / [0.85 \times 21 \times 1200] = 81.3 \text{ mm} > h_f$

T beam design is required and $\phi = 0.9$ is assumed.

$A_{sf} = 0.85 f'_c (b - b_w) h_f / f_y = 0.85 \times 21 (1200 - 250) \times 75 / 420 = 2948 \text{ mm}^2$

$\phi M_{n1} = \phi A_{sf} f_y (d - h_f/2) = 0.9 [2948 \times 420 (500 - 75/2) \times 10^{-6}] = 515.4 \text{ kNm}$

$\phi M_{n2} = M_n - \phi M_{n1} = 725 - 515.4 = 209.6 \text{ kNm}$

Assume $a = 100 \text{ mm}$

$A_s - A_{sf} = \phi M_{n2} / \phi f_y (d - a/2) = 209.6 \times 106 / [0.9 \times 420 \times (500 - 50)] = 1232 \text{ mm}^2$

Check $a = (A_s - A_{sf}) f_y / (0.85 f'_c b_w) = 1232 \times 420 / [0.85 \times 21 \times 275] = 105.4 \text{ mm} \approx 100 \text{ mm}$ ok

$A_s = A_{sf} + A_s - A_{sf} = 2948 + 1232 = 4181 \text{ mm}^2$

Check: $c = a / \beta_1 = 105.4 / 0.85 = 124 \text{ mm}$

$c / d_t = 124 / 500 = 0.248 < 0.375$, so $\epsilon_t > (\epsilon_y + 0.003) = 0.005 \rightarrow \phi = 0.9$

Design is satisfactory.