

Engineering Mechanics I

Chapter -6-Structural Analysis



- To show how to determine the forces in the members of a truss using the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.





- Simple Trusses
- The Method of Joints
- Zero-Force Members
- The Method of Sections

6.1 Simple Trusses

- A truss is a structure composed of slender members joined together at their end points.
- Joint connections are formed by bolting or welding the ends of the members to a common plate, called a gusset plate, or by simply passing a large bolt or pin through each of the members





6.1 Simple Trusses

Planar Trusses

- Planar trusses lie on a single plane and are used to support roofs and bridges.
- The truss ABCD shows a typical roof-supporting truss.
- Roof load is transmitted to the truss at joints by means of a series of purlins, such as DD'





Planar Trusses

• The analysis of the forces developed in the truss members is 2D.





Planar Trusses

- For a bridge, the load on the deck is first transmitted to the stringers, then to the floor beams, and finally to the joints B, C and D of the two supporting trusses.
- Like the roof truss, the bridge truss loading is also coplanar.





6.1 Simple Trusses

Planar Trusses

- When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, Eg: joint E.
- This type of support allows freedom for expansion or contraction of the members due to temperature or application of loads.





Assumptions for Design

- 1. "All loadings are applied at the joint".
- Assumption true for most applications of bridge and roof trusses.
- Weight of the members neglected since forces supported by the members are large in comparison.
- If member's weight is considered, apply it as a vertical force, half of the magnitude applied at each end of the member.



Assumptions for Design

- 2. "The members are joined together by smooth pins".
- Assumption true when bolted or welded joints are used, provided the center lines of the joining members are concurrent.









6.1 Simple Trusses

Simple Truss

- To prevent collapse, the form of a truss must be rigid.
- The four bar shape ABCD will collapse unless a diagonal member AC is added for support.
- The simplest form that is rigid or stable is a triangle.





6.1 Simple Trusses

Simple Truss

• A simple truss is constructed starting with a basic triangular element such as ABC and connecting two members (AD and BD) to form an additional element.







- For design analysis of a truss, we need to obtain the force in each of the members.
- Considering the FBD, the forces in the members are internal forces and could not be obtained from an equilibrium analysis.
- Considering the equilibrium of a joint of the truss, a member force becomes an external force on the joint's FBD and equations of equilibrium can be applied.
- This forms the basis for the method of joints.



- Truss members are all straight two force members lying in the same plane.
- The force system acting at each joint is coplanar and concurrent.
- Rotational or moment equilibrium is automatically satisfied at the pin.
- $\Sigma F_x = 0$ and $\Sigma F_y = 0$ must be satisfied for equilibrium.



Method of Joints

- Draw FBD.
- Line of action of each member force acting on the joint is specified from the geometry of the truss since the force in a member passes along the axis of the member.

Example

- Consider pin at joint B.
- Three forces: 500N force and forces exerted by members BA and BC.



6.2 The Method of Joints

- \mathbf{F}_{BA} is "pulling" on the pin, meaning the member BA is in tension.
- \mathbf{F}_{BC} is "pushing" on the pin, meaning the member BC is in compression.
- The pushing and pulling indicates the effect of the member being either in tension or compression.





Determining the Correct Sense of the Unknown Member

 Always assume the unknown member forces acting on the joint's FBD to be in tension.

- The numerical solution of the equilibrium will yield positive scalars for members in tension and negative scalars for members in compression.

- Use the correct magnitude and sense of the unknown member on subsequent FBD.



Determining the Correct Sense of the Unknown Member

• The correct sense of a direction of an unknown force can be determined by inspection.

- $\mathbf{F}_{\rm BC}$ must push on the pin (compression) since its horizontal component must balance the 500N force.

- \mathbf{F}_{BA} is a tensile force since it balances the vertical component of \mathbf{F}_{BC} .



Determining the Correct Sense of the Unknown Member

- The correct sense of a direction of an unknown force can be determined by inspection.
 - In more complicated problems, the sense of the member can be assumed.
 - A positive answer indicates that the assumed sense is correct and a negative answer indicates that the assumed sense must be reversed.



Procedure for Analysis

- Draw the FBD of a joint having at least one known force and at most two unknown forces.
- If this joint is at one of the supports, determine the external reactions at the truss support.
- Use one of two methods for determining the correct sense of the member.
- Orient the x and y axes so that the forces on the FBD can be easily resolved into x and y components.



Procedure for Analysis

- Apply $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
- Solve for unknown members forces and verify their correct sense.
- Continue to analyze each of the other joints.
- Once the force in a member is found from the analysis of the joint at one of its end, the result is used to analyze the forces acting on the other end.



Example 6.1

Determine the force in each member of the truss and indicate whether the members are in tension or compression.





Solution

- Two unknown member forces at joint B.
- One unknown reaction force at joint C.
- Two unknown member forces and two unknown reaction forces at point A.





Solution

Joint C

+ → Σ
$$F_x = 0;$$

- $F_{CA} + 707.1\cos 45^\circ N = 0$
 $F_{CA} = 500N(T)$
+ ↑ Σ $F_y = 0;$
 $C_y - 707.1\sin 45^\circ N = 0$
 $C_y = 500N$



Solution Joint A $+ \rightarrow \sum F_x = 0;$ $500N - A_x = 0$ $A_x = 500N$ $+ \uparrow \sum F_y = 0;$ $500N - A_y = 0$ $A_y = 500N$ (d)



Solution

- FBD of each pin shows the effect of all the connected members and external forces applied to the pin.
- FBD of each member shows only the effect of the end pins on the member.





Example 6.2

Determine the force in each member of the truss, and state if the members are in tension or compression.







Method of Joints: We will begin by analyzing the equilibrium of joint D, and then proceed to analyze joints C and D.

Joint D: From ge	cometry, $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^{\circ}$. Thus, from the	free - body diagram in Fig. a,
$\stackrel{+}{\rightarrow}\Sigma F_x = 0;$	$600 - F_{DC} \sin 26.57^\circ = 0$	
	$F_{DC} = 1341.64 \text{ N} = 1.34 \text{ kN}$ (C)	Ans.
$+\uparrow \Sigma F_y = 0;$	$1341.64\cos 26.57^\circ - F_{DE} = 0$	
	$F_{DE} = 1200 \mathrm{N} = 1.20 \mathrm{kN} \mathrm{(T)}$	Ans.



Joint C: From the free - body diagram in Fig. b, $+ \oint \Sigma F_{x'} = 0; -F_{CE} \cos 26.57^{\circ} = 0$ $F_{CE} = 0$ $+ \sum F_{y'} = 0; F_{CB} - 1341.64 = 0$ $F_{CB} = 1341.64 N = 1.34 kN (C)$ Ans.

Joint E: From the free - body diagram in Fig. c,

$+\Sigma F_x = 0,$	$900 - F_{EB} \sin 45^\circ = 0$	
	$F_{EB} = 1272.79 \mathrm{N} = 1.27 \mathrm{kN}$ (C)	Ans.
$+\uparrow\Sigma F_y=0;$	$1200 + 1272.79\cos 45^\circ - F_{EA} = 0$	
	$F_{EA} = 2100 \mathrm{N} = 2.10 \mathrm{kN} \mathrm{(T)}$	Ans.

Note. The equilibrium analysis of joint A can be used to determine the components of support reaction at A.



Example 6.3

Determine the forces acting in all the members of the truss.





Solution

- Two unknowns at each joint.
- Support reactions on the truss must be determined.











Solution

• From the FBD of joint B, sum the forces in the horizontal direction.

 $F_{BA} = 0.776 kN (C)$



Example 6.4

Determine the force in each member of the truss. Indicate whether the members are in tension or compression.







Problem 6.5

Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

By inspection of joint D:

FBDs Joints:

$$A \xrightarrow{F_{AB}} F_{AC}$$

$$\Rightarrow \Sigma F_{y} = 0:9 \text{ kN} - \frac{4}{5} F_{AB} = 0$$

$$F_{AB} = 11.25 \text{ kN C}$$

$$\Rightarrow \Sigma F_{x} = 0: F_{AC} - \frac{3}{5} F_{AB} = 0$$

$$F_{AC} = 6.75 \text{ kN T}$$

$$9 \text{ kN}$$

 $F_{FH} = 11.25 \text{ kN C}$ $F_{GH} = 6.75 \text{ kN T}$ $F_{EG} = 6.75 \text{ kN T}$ $F_{FG} = 6.00 \text{ kN T}$ $F_{EF} = 3.75 \text{ kN C}$ $F_{DF} = 9.00 \text{ kN T}$

truss during construction and to provide support if the applied loading is changed.

6.3 Zero-Force Members

- Consider FBD of joint D
- DC and DE are zero-force members.
- As a general rule, if only two members form a truss joint and no external load or support reaction is applied to the F_{DE} joint, the members must be zero-force members.

 $+ \searrow \Sigma F_{y} = 0; F_{DC} \sin \theta = 0; F_{DC} = 0 \text{ since } \sin \theta \neq 0$ $+ \swarrow \Sigma F_x = 0; F_{DE} + 0 = 0; F_{DE} = 0$ (c)

• The load on the truss shown in fig (a) is therefore supported by only five members as shown in fig (d)

In general, if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint
The truss shown is suitable for supporting the load P

(d)

Example 6.6

Using the method of joints, determine all the zero-force members of the Fink roof truss. Assume all joints are pin connected.

Solution

 F_{HC} satisfy $\Sigma F_y = 0$ and therefore HC is not a zero-force member.

- It can be seen that equilibrium requires the member in tension (T) be subjected to a pull and the member in compression (C) be subjected to a push
- Method of section can be used to cut or section members of an entire truss
- Apply equations of equilibrium on that part to determine the members

- Consider the truss shown
- To determine the force in the member GC, section aa would be considered

(c)

- Consider the FBD .
- Note the line of action of each member force is specified from the geometry of the truss.
- Member forces acting on one part of the truss are equal and opposite to those acting on the other part – Newton's Law.

- Members assumed to be in tension (BC and GC) are subjected to a pull whereas the member in compression (GF) is subjected to a push.
- Apply equations of equilibrium.

Determining the Correct Sense of the Unknown Member

- Always assume the unknown member forces in the cut section are in tension.
 - The numerical solution of the equilibrium will yield positive scalars for members in tension and negative scalars for members in compression.

Determining the Correct Sense of the Unknown Member

- The correct sense of a direction of an unknown force can be determined by inspection
 - In more complicated problems, the sense of the member can be assumed
 - A positive answer indicates that the assumed sense is correct and a negative answer indicates that the assumed sense must be reversed

Procedure for Analysis

- Draw the FBD of a joint having at least one known force and at most two unknown forces.
- If this joint is at one of the supports, it ma be necessary to know the external reactions at the truss support.
- Use one of two methods for determining the correct sense of the member.
- Orient the x and y axes so that the forces on the FBD can be easily resolved into x and y components.

Procedure for Analysis

Free-Body Diagram

- Decide how to cut or session the truss through the members where forces are to be determined.
- Before isolating the appropriate section, determine the truss's external reactions.
- Use the equilibrium equations to solve for member forces at the cut session.

Procedure for Analysis

Free-Body Diagram

- Draw the FBD of that part of the sectioned truss which has the least number of forces acting on it.
- Use one of the two methods for establishing the sense of an unknown member force.

Procedure for Analysis

Equations of Equilibrium

- Moments are summed about a point that lies at the intersection of lines of action of the two unknown forces.
- The third unknown force is determined directly from moment equation.
- If two of the unknown forces are parallel, forces may be summed perpendicular to the direction of these unknowns to determine the third unknown force.

Example 6.7

Determine the force in members GE, GC, and BC of the truss. Indicate whether the members are in tension or compression.

Solution

- Choose section aa since it cuts through the three members
- FBD of the entire truss

6.2 The Method of Joints

Solution

$$\begin{split} &+ \rightarrow \sum F_x = 0;400N - A_x = 0\\ &A_x = 400N\\ &\sum M_A = 0;\\ &-1200N(8m) - 400N(3m) + D_y(12m) = 0\\ &D_y = 900N\\ &+ \uparrow \sum F_y = 0;\\ &A_y - 1200N + 900N = 0\\ &A_y = 300N \end{split}$$

Solution

FBD of the sectioned truss

6.2 The Method of Joints

Solution

$$\begin{split} \sum M_G &= 0; \\ &- 300N(4m) - 400N(3m) + F_{BC}(3m) = 0 \\ F_{BC} &= 800N(T) \\ \sum M_C &= 0; \\ &- 300N(8m) + F_{GE}(3m) = 0 \\ F_{GE} &= 800N(C) \\ &+ \uparrow \sum F_y = 0; \\ 300N - \frac{3}{5}F_{GC} &= 0 \\ F_{GC} &= 500N(T) \end{split}$$

Example 6.8

Determine the force in member CF of the bridge truss. Indicate whether the member are in tension or compression. Assume each member is pin connected.

Solution FBD of the entire truss

Solution

FBD of the sectioned truss

• Three unknown F_{FG}, F_{CF}, F_{CD}

Example 6.9

Determine the force in member EB of the roof truss. Indicate whether the member are in tension or compression. 1000 N

Solution

- Force system is concurrent.
- Sectioned FBD is same as the FBD for the pin at E (method of joints).

Solution

$$\begin{split} & \sum M_B = 0; \\ & 1000N(4m) - 3000N(2m) - 4000N(4m) + F_{ED} \sin 30^{\circ}(4m) = 0 \\ & F_{ED} = 3000N(C) \\ & + \rightarrow \sum F_x = 0; \\ & F_{EF} \cos^{\circ} - 3000 \cos 30^{\circ} N = 0 \\ & F_{EF} = 3000N(C) \\ & + \uparrow \sum F_y = 0; \\ & 2(3000 \sin 30^{\circ} N) - 1000N - F_{EB} = 0 \\ & F_{EB} = 2000N(T) \end{split}$$

Homework 1

Determine the force in members *HI*, *FI*, and *EF* of the truss, and state if the members are in tension or compression.

Determine the force in members BG, BC, and HG of the truss and state if the members are in tension or compression.

Determine the force in members *IC* and *CG* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

