8 Short Columns

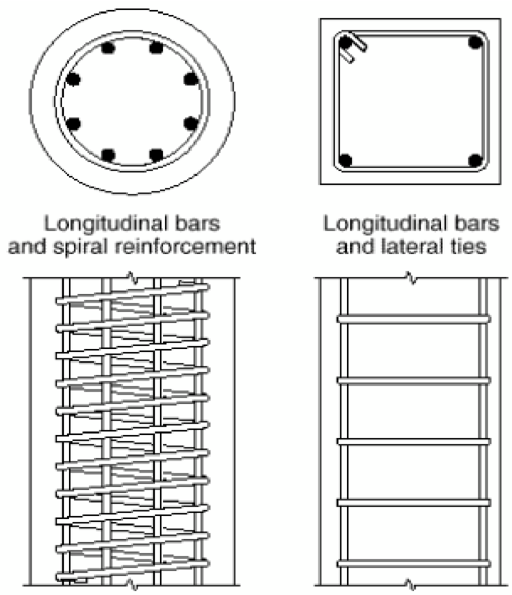
Columns, generally referred as *compression members*, are members that carry loads chiefly in compression. Usually columns carry bending moments as well, about one or both axes of the cross section.

The most common types of RC compression members in use are:

1. Members reinforced with longitudinal bars and lateral ties.
2. Members reinforced with longitudinal bars and continuous spirals.

Other type is the *composite compression members* reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement.

The main reinforcement in columns is longitudinal, parallel to the direction of load, and consists of bars arranged in a square, rectangular, or circular pattern, as shown in Fig. below:



According to ACI Code 10.9.2, a minimum of 4 longitudinal bars is required when the bars are enclosed by spaced rectangular or circular ties, and a minimum of 6 bars must be used when the longitudinal bars are enclosed by a continuous spiral.

According to ACI Code 10.9.1, the ratio of longitudinal steel area A_{st} to gross concrete cross section A_g is in the range from **0.01** to **0.08**.

The ***lower limit*** ($A_{st} / A_g = 0.01$) ensures resistance to bending moment not accounted for in the analysis, and to reduce the effects of creep and shrinkage of concrete under sustained compression. The ***upper limit*** ($A_{st} / A_g = 0.08$) is set for economy and to avoid difficulties owing to congestion of reinforcement.

Columns may be divided into two categories:
Short columns: In which the strength is governed by the strength of the materials and the geometry of the cross section.
Slender columns: In which the strength may be significantly reduced by lateral deflections.

Behavior of Short, Axially Loaded Compression Members

The nominal strength of an axially loaded RC column can be found, recognizing the nonlinear response of both materials, by summing the contribution of steel and concrete:

$$P_n = 0.85f'_c\,(A_g - A_{st}) + A_{st}\,f_y$$

According to ACI Code 10.3.6, the equation should introduce certain reduction factors [φ], and further limitations to allow for accidental eccentricities not considered in design, as follows:

Design axial strength:

For spirally reinforced columns:

$$\varphi P_{n,max} = 0.85\,\varphi\,[0.85f'_c\,(A_g - A_{st}) + A_{st}\,f_y] \qquad \varphi = 0.75$$

For tied columns:

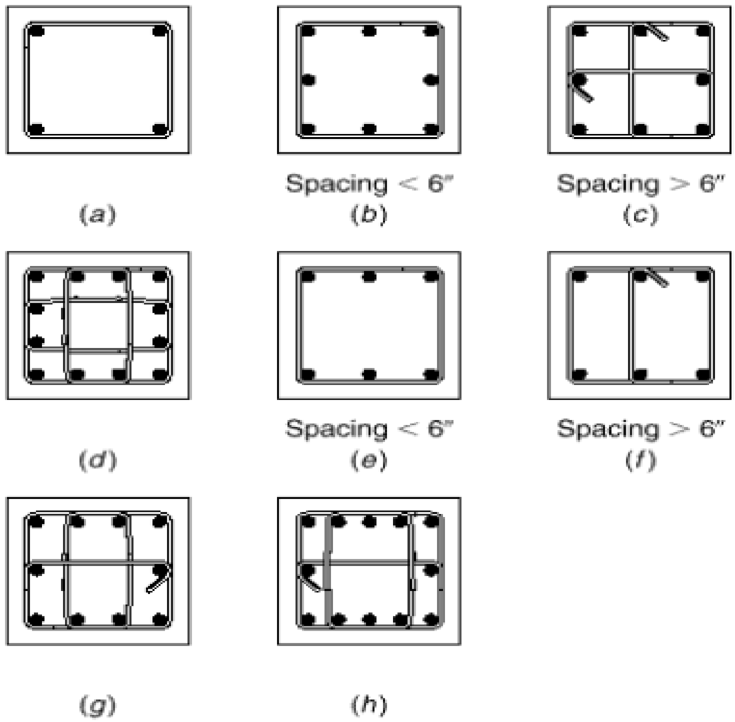
$$\varphi P_{n,max} = 0.80\,\varphi\,[0.85f'_c\,(A_g - A_{st}) + A_{st}\,f_y] \qquad \varphi = 0.65$$

Lateral Ties and Spirals

Lateral ties and spirals serve several functions:

- Hold the longitudinal bars in position in the forms while concrete is being placed.
- Prevent the longitudinal bars from buckling outward by bursting the concrete cover.

Accordingly, ties and spirals should be closely spaced, and number of ties may be placed in the same plane. (See Figs c, d, f, g and h)
Fig. below shows column cross sections frequently found in buildings and bridges.
In Figs *a* to *d*, columns with large axial forces and small moments are shown in which bars are spaced uniformly around the perimeter.
In Figs *e* to *h*, columns with large bending moments are shown in which the bars are concentrated at the faces of high compression or tension.



For tied columns, according ACI Code 7.10.5:
The **ties** shall be **at least No.10** in size for longitudinal bars up to No.32, and **at least No.13** in size for Nos. 36, 43, and 57.
The spacing of ties ≤ **(16d_b, 48d_{tie}, least dimension of the column)**
The ties shall be so arranged as shown in Fig below:

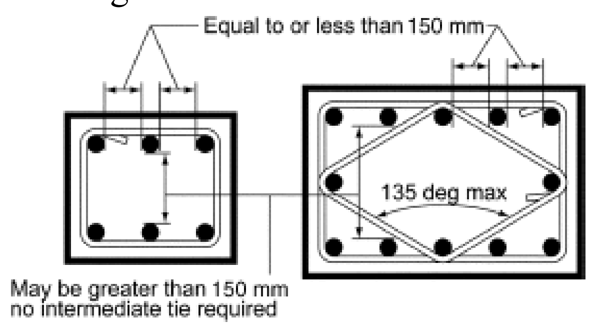


Fig. R7.10.5—Sketch to clarify measurements between laterally supported column bars.

For spirally reinforced columns, according to ACI Code 7.10.4, **spirals** shall consist of a continuous bar or wire **≥10 mm in diameter**, and the clear spacing between turns of spiral (*pitch*) must be (**25 mm ≤ *pitch* ≤ 75 mm**). In addition, according to ACI Code 10.9.3, the *volumetric* ratio of spiral reinforcement ($\boldsymbol{\rho_s}$) shall not be less than: (f_y must be ≤ 420 MPa)

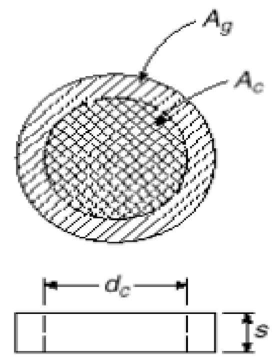
$$\rho_s = 0.45[(A_g / A_{ch}) - 1](f'_c / f_y)$$

Where

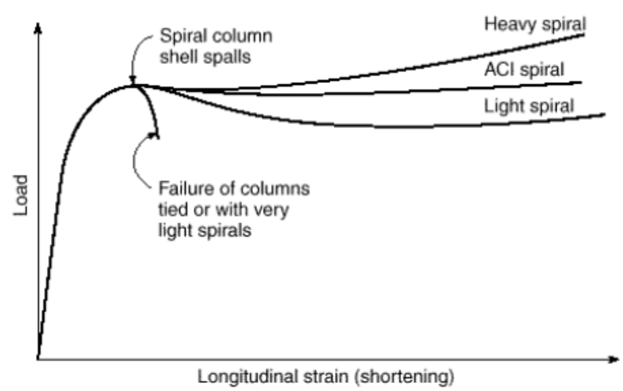
$$\rho_s = 4A_{sp} / (d_c\, s)$$

The volumetric ratio = ratio of (*vol. of spiral steel / vol. of core concrete*)

$\rho_s = (2\pi\, d_c\, A_{sp} / 2) / (\pi\, d_c^2\, s / 4) = 4A_{sp} / (d_c\, s)$



This is imposed to improve the structural performance w.r.t. both ultimate load and type of failure, compared with an otherwise identical tied column.



<u>Example 1:</u> For **400 mm** circular column, check spiral reinforcement **No.10 @ 50 mm** pitch. Concrete **cover = 40 mm**, f'_c **= 28 MPa** and f_y **= 420 MPa.**

<u>Solution:</u>

$\rho_s = 4A_{sp} / (d_c\, s) = 4\times71 / (320\times50) = 0.0178$

$\rho_s = 0.45[(A_g / A_{ch}) - 1](f'_c / f_y)$

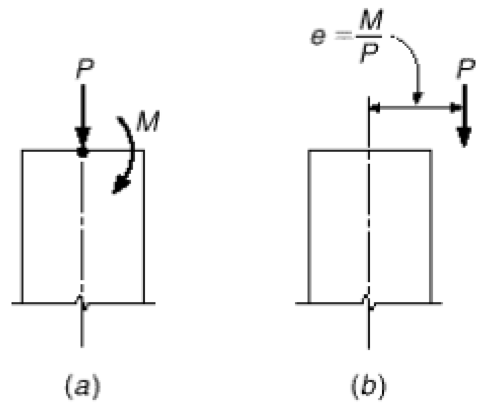
$= 0.45[(\pi\times200^2 / \pi\times160^2) - 1](28/420) = 0.0169$

0.0169 < 0.0178 **OK**

Compression plus Bending of Rectangular Columns

Members that are axially, i.e., concentrically, compressed occur rarely, if ever, in buildings and other structures. Bending moments are caused by continuity, by transverse loads such as wind forces, by loads carried eccentrically on column brackets, or by imperfections of construction. For this reason, almost all columns are designed for simultaneous compression and bending.

When a member is subjected to combined axial compression ***P*** and moment ***M***, see Fig. a, it is convenient to replace the axial load and moment with an equal load ***P*** applied at eccentricity $\boldsymbol{e = M / P}$, (Fig. b)



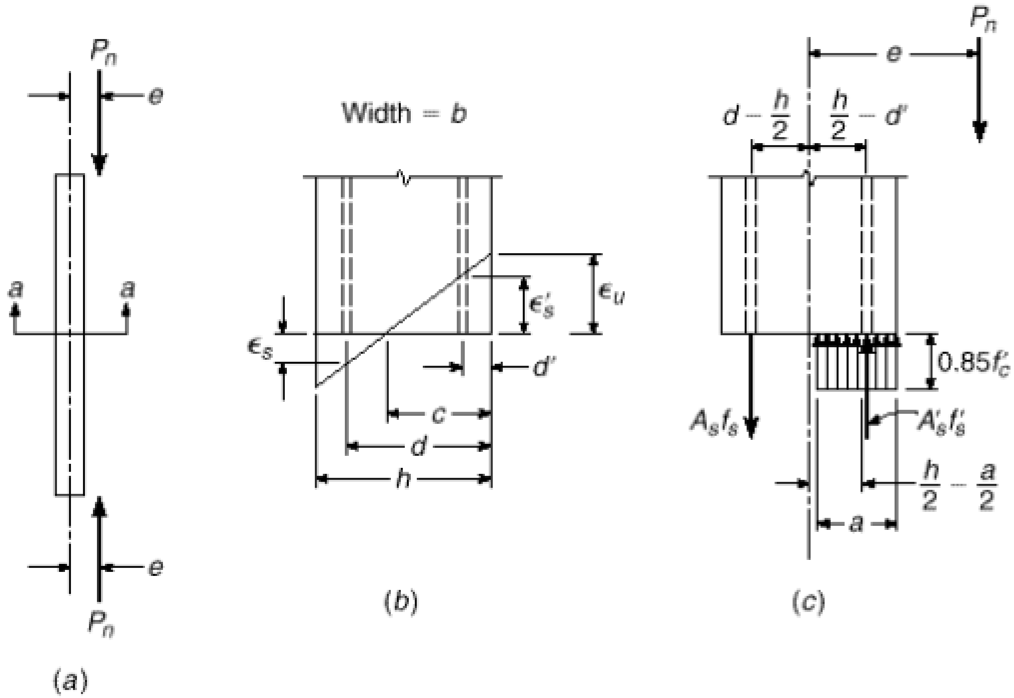
Columns having small ***e*** are characterized by compression over the entire concrete section. They will fail by concrete crushing and yielding of steel in compression.
Columns having large ***e*** are characterized by tension over at least part of the concrete section. They will fail by tensile yielding of steel on the side farthest from the load.

Design of columns is based on factored load, which must not exceed the design strength:

$$\varphi M_n \geq M_u \qquad\qquad \varphi P_n \geq P_u$$

Interaction Diagrams

Fig. (a) below shows a member loaded parallel to its axis by a compressive force $\boldsymbol{P_n}$ at an eccentricity $\boldsymbol{e}$ measured from the centerline. Fig. (b) shows the strain distribution at section a-a, at failure. The corresponding stresses and forces are shown in Fig. (c).



From equilibrium:

$\boldsymbol{\Sigma F = 0}$, $(F_{external} = F_{internal})$

$$\boldsymbol{P_n = 0.85f'_c\, ab + A'_s\, f'_s - A_s\, f_s} \qquad \ldots\ldots\ \textbf{(1)}$$

$\boldsymbol{\Sigma M_{centerline} = 0}$, $(M_{external} = M_{internal})$

$$\boldsymbol{M_n = P_n e = 0.85f'_c\, ab\, (h/2 - a/2) + A'_s\, f'_s(h/2 - d') - A_s\, f_s\, (d - h/2)} \quad \ldots.\ \textbf{(2)}$$

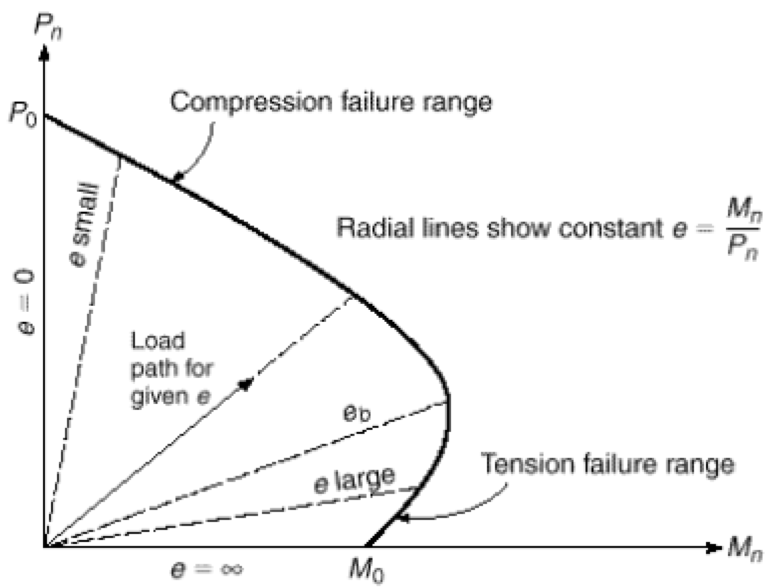
For a given $\boldsymbol{e}$ determined from frame analysis ($\boldsymbol{e = M_u / P_u}$) it is possible to solve Eqs. 1 and 2 for $\boldsymbol{P_n}$ and $\boldsymbol{M_n}$ as follows:

*From strain diagram, with $\varepsilon_u = 0.003$, $\boldsymbol{f'_s}$, $\boldsymbol{f_s}$, and **a** can be expressed in terms of a single unknown **c**. The result is that the two equations contain only two unknowns, $\boldsymbol{P_n}$ and **c** and can be solved for them simultaneously.*

However, to do this in practice would be complicated algebraically.

A better approach is to construct a ***strength interaction diagram*** defining the failure load and failure moment for a given column for the full range of eccentricities from **0** to ∞.

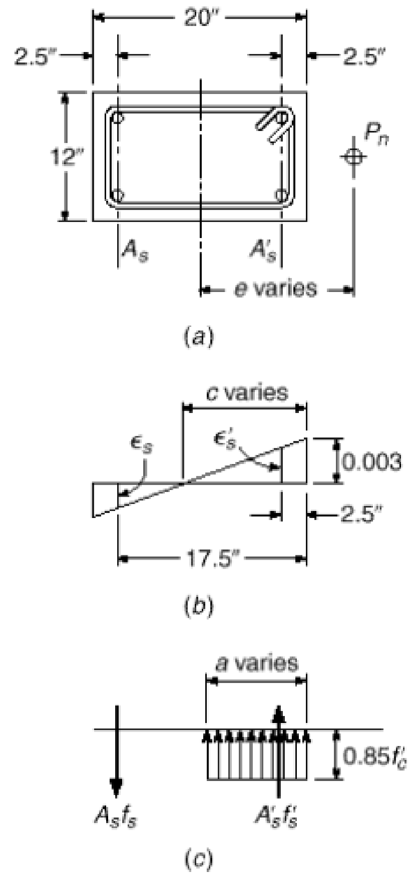
For any eccentricity e, there is a unique pair of P_n and M_n. That pair can be plotted as a point on a graph relating P_n and M_n, see Fig. below. A series of such calculations, corresponding to different eccentricities will result in a curve having a shape typically as shown below:



Interaction diagram for nominal column strength in combined bending and axial load.

Any radial line represents a particular $e = M / P$
The vertical axis: ($e = 0$), and P_o = capacity of concentrically loaded column.
The horizontal axis: ($e = \infty$), and M_o = pure bending capacity of column.
e small: produces failure governed by concrete compression.
e large: produces failure governed by yielding of tension steel.
e_b : produces "***balanced failure***", with P_b and M_b acting in combination. The concrete reaches its limit strain ε_u at the same instant that tensile steel reaches yield strain. This point on the interaction diagram is the dividing point between compression failure (***e*** small) and tension failure (***e*** large).

Example 2: (*Construction of Interaction Diagram*)
A **300×500 mm** column is reinforced with **4 No.29** bars of area **645 mm²** each, one in each corner as shown in Fig. f'_c **= 28 MPa and** f_y **= 420 MPa**.
Determine:
(a) P_b and M_b , and e_b.
(b) The load and moment for a point in the tension failure region.
(c) The load and moment for a point in the compression failure region.
(d) P_o for $e = 0$.
(e) Sketch the 'Interaction Diagram' of this column,
(f) Design the transverse reinforcement.



2.5" = 65 mm
17.5" = 435 mm
12" = 300 mm
20" = 500 mm

a) From strain diagram $c = c_b = d[\varepsilon_u /(\varepsilon_u + \varepsilon_y)]$
$= 435[0.003/0.0051] = 256$ mm
$a = \beta_1 c = 0.85(256) = 218$ mm
$f'_s = E_s\,\varepsilon_u\,(c - d')/c = 448$ MPa > 420 MPa
$C = 0.85\times 28\times 218\times 300\times 10^{-3} = 1557$ kN
$\boldsymbol{P_b = 0.85f'_c\,ab + A'_s\,f'_s - A_s\,f_s}$
$= 1557 + 1290\times 420\times 10^{-3} - 1290\times 420\times 10^{-3} =$ **<u>1557 kN</u>**
$\boldsymbol{M_b = 0.85f'_c\,ab\,(h/2 - a/2) + A'_s\,f'_s(h/2 - d') + A_s\,f_s\,(d - h/2)}$
$= 1557(250 - 109) + 1290\times 420(250 - 65) + 1290\times 420(435 - 250)$
$= 420\times 10^3$ kN-mm = **<u>420 kNm</u>**
$\boldsymbol{e_b}$ **=** **<u>241 mm</u>**

b) Any $c < c_b = 256$ mm will give a point in tension failure region: $\boldsymbol{e > e_b}$
For e.g., choose $\boldsymbol{c}$ **= 125 mm.**
$f'_s = 200{,}000\times 0.003(125 - 65)/125 = 288$ MPa
$a = \beta_1 c = 0.85(125) = 106$ mm
$C = 0.85\times 28\times 106\times 300\times 10^{-3} = 757$ kN
$\boldsymbol{P_n = 0.85f'_c\,ab + A'_s\,f'_s - A_s\,f_s}$
$= 757 + 1290\times 288\times 10^{-3} - 1290\times 420\times 10^{-3} =$ **<u>587 kN</u>**

$\boldsymbol{M_n = 0.85f'_c\, ab\, (h/2 - a/2) + A'_s\, f'_s(h/2 - d') - A_s\, f_s\, (d - h/2)}$

$= 757(250 - 53) + 1290\times288(250 - 65) + 1290\times420(435 - 250)$

$= 318\times10^3$ kN-mm = **<u>318 kNm</u>**

***e* = <u>542 mm</u>**

c) Any $c > c_b$ =256 mm will give a point in compression failure region: $\boldsymbol{e < e_b}$

For e.g., choose $\boldsymbol{c = 460}$ **mm.**

$f_s = 200{,}000 \times 0.003(435 - 460)/460 = -33$ MPa (*indicates A_s is in comp.*)

$f'_s = 200{,}000 \times 0.003(460 - 65)/460 = 515$ MPa > 420

$a = \beta_1 c = 0.85(460) = 391$ mm

$C = 0.85\times28 \times391\times300\times10^{-3} = 2792$ kN

$\boldsymbol{P_n = 0.85f'_c\, ab + A'_s\, f'_s - A_s\, f_s}$

$= 2792 + 1290\times420\times10^{-3} - 1290\times(-33)\times10^{-3}$ = **<u>3376 kN</u>**

$\boldsymbol{M_n = 0.85f'_c\, ab\, (h/2 - a/2) + A'_s\, f'_s(h/2 - d') - A_s\, f_s\, (d - h/2)}$

$= 2792(250 - 196) + 1290\times420(250 - 65) + 1290\times(-33)(435 - 250)$

$= 243\times10^3$ kN-mm = **<u>243 kNm</u>**

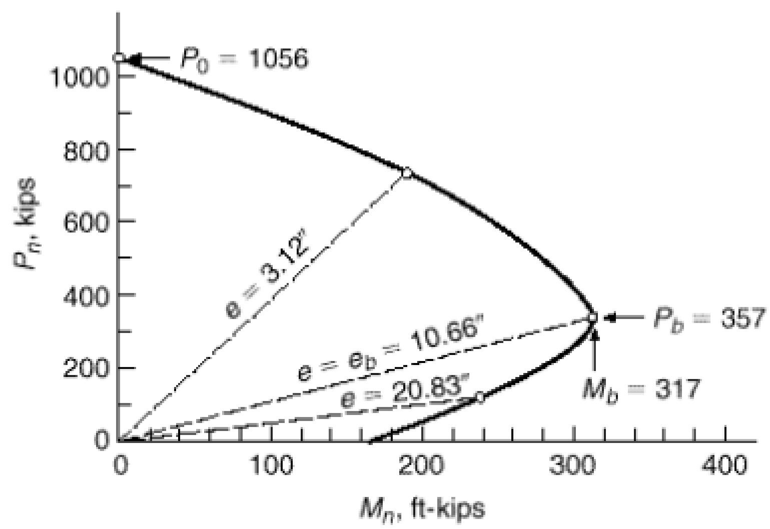
***e* = <u>72 mm</u>**

d) The axial strength of concentrically loaded column ($c = \infty$, $\boldsymbol{e = 0}$):

$\boldsymbol{P_o = 0.85f'_c\, (A_g - A_{st}) + A_{st}\, f_y}$ (*neglect A_{st} deduction, error is only 1%*)

$= [0.85\times28 \times300\times500 + 2580\times420]\times10^{-3}$ = **<u>4654 kN</u>**

e) The interaction diagram can be constructed:



f) Minimum permitted bar size is No.10 bar.

Spacing is min of ($48d_{tie}$, $16d_b$, least dim of column)

(48×9.5, 16×28.7, 300) mm = (456, 459, <u>300</u>) mm

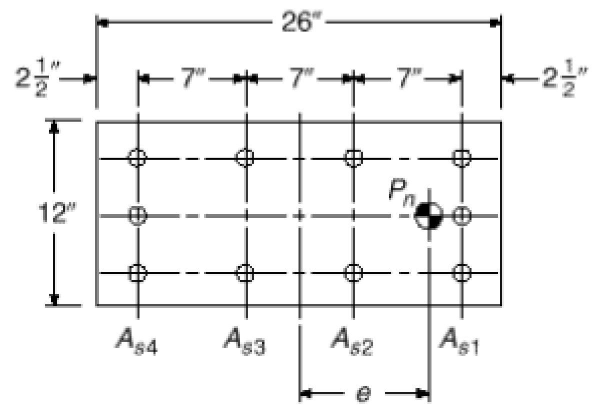
<u>Use No.10 ties @ 300 mm</u>

Distributed Reinforcement

It is often advantageous to place steel uniformly around the perimeter when axial compression is predominant (small e). The intermediate bars will be stressed below yield point. This situation can be analyzed based on strain compatibility.

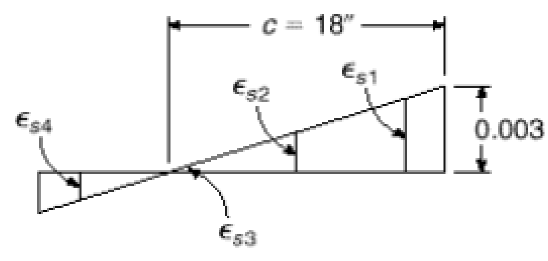
Example 3:

The column below (300×670 mm) is reinforced with **10 No.36** bars distributed around the perimeter. Load P_u will be applied with eccentricity e about the strong axis. f'_c **= 42 MPa and** f_y **= 550 MPa**. Find the load and moment corresponding to a failure point with NA c **= 460 mm** from the right face. [*2½" = 65 mm, 7" = 180 mm, 12" = 300 mm, 26" = 670 mm*]

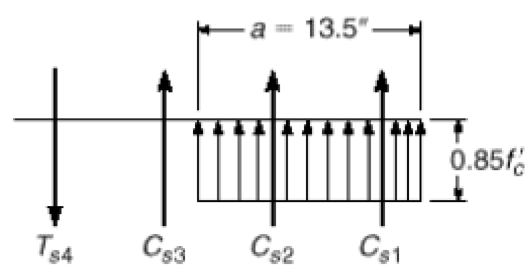


(a)

Solution: c = 460 mm(18"), β_1 = 0.75(check this!), $a = \beta_1 c$ =345 mm(13.5")



(b)



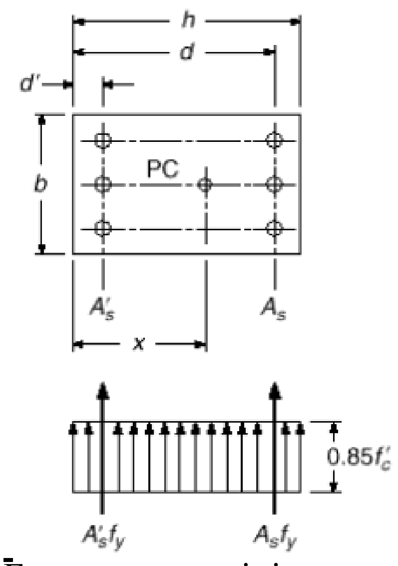
(c)

From strain diagram (fig. b): (ε_u = 0.003, E_s = 200,000 MPa)
$\varepsilon_{s1} = 0.003[(c - d')/c] = 0.003[(460 - 65)/460] = 0.00258$
$\varepsilon_{s1} = 0.00258$ $f_{s1} = E_s\,\varepsilon_{s1} = 516$ MPa compression
$\varepsilon_{s2} = 0.00140$ $f_{s2} = E_s\,\varepsilon_{s2} = 280$ MPa compression
$\varepsilon_{s3} = 0.00023$ $f_{s3} = E_s\,\varepsilon_{s3} = 46$ MPa compression
$\varepsilon_{s4} = 0.00095$ $f_{s4} = E_s\,\varepsilon_{s4} = 190$ MPa tension

$C = [0.85\times42\times345\times300]\times10^{-3} = 3695$ kN concrete compression
From fig. c:
$C_{s1} = A_{s1} f_{s1}$, $A_{s1} = 3$ No.36 $= 3\times1006 = 3018$ mm^2
$C_{s1} = 3018\times516\times10^{-3} = 1557$ kN
$C_{s2} = 2012\times280\times10^{-3} = 563$ kN
$C_{s3} = 2012\times46\times10^{-3} = 92$ kN
$T_{s4} = 3018\times190\times10^{-3} = 573$ kN

$\Sigma F = 0$; $P_n = 3695 + 1557 + 563 + 92 - 573 =$ **5334 kN**
$\Sigma M = 0$; $M_n = 3695(335 - 172.5) + 1557(335 - 65) + 563(335 - 245) - 92(335 - 245) + 573(335 - 65) = 1218000$ kN-mm = **1218 kN-m**
$e = 1218 / 5334 = 0.228$ m = **228 mm**

Note: Unsymmetrical Reinforcement



$$x = \frac{0.85f'_c bh^2/2 + A_s f_y d + A'_s f_y d'}{0.85f'_c bh + A_s f_y + A'_s f_y}$$

For some cases it is more economical to use unsymmetrical pattern of bars, with most of the bars on the tension side. Such columns can be analyzed by the same strain compatibility approach as described in the previous example. However, for such columns to be loaded concentrically, the load must pass thru a point known as *plastic centroid* (*PC*). It is defined as $x = \Sigma F\bar{y} / \Sigma F$. See Eq. above. Eccentricity must be measured w.r.t. the *PC*.

Circular Columns

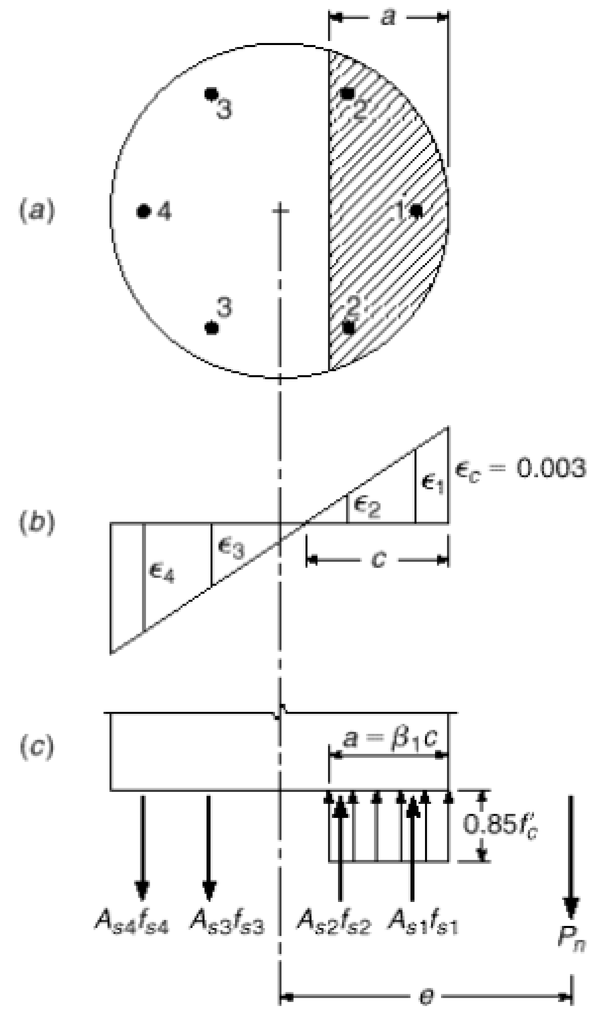
Spirally reinforced columns show greater ductility than tied columns, esp. when load eccentricities are small. Also, the max design load for axially loaded members is larger for spirally reinforced columns than tied columns. For these reasons, the ACI Code provides φ **= 0.75** for spiral columns, compared with φ **= 0.65** for tied columns.

Fig. (a) below shows a cross section of a spirally reinforced column.

Fig. (b) shows the strain distribution.

Fig. (c) shows the internal forces.

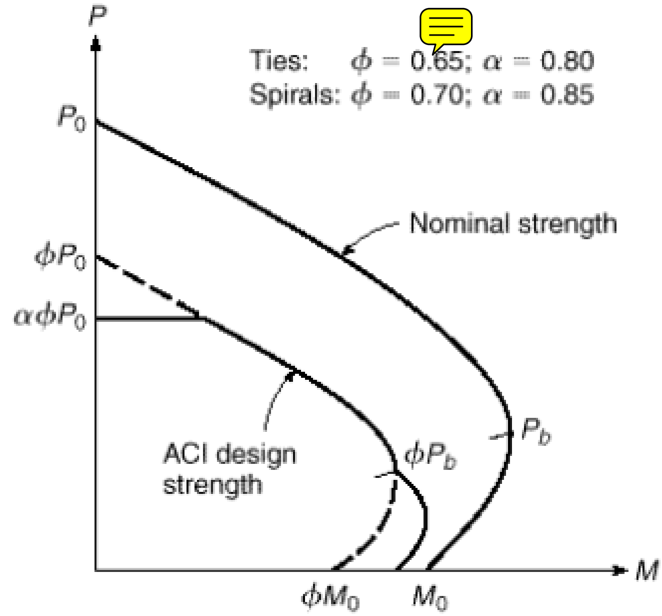
Calculations for P_n and M_n can be carried out exactly as in the previous example, except that the concrete compression zone has the shape of a segment of a circle.



Design or analysis of spirally reinforced columns is usually carried by means of ***design aids***. This is also true for tied columns.

ACI Code Provisions for Column Design

ACI Code provides basic reduction factors: $\boldsymbol{\varphi} = \mathbf{0.75}$ for spiral columns, and $\boldsymbol{\varphi} = \mathbf{0.65}$ for tied columns.
The effects of these safety provisions are shown in Fig. below:



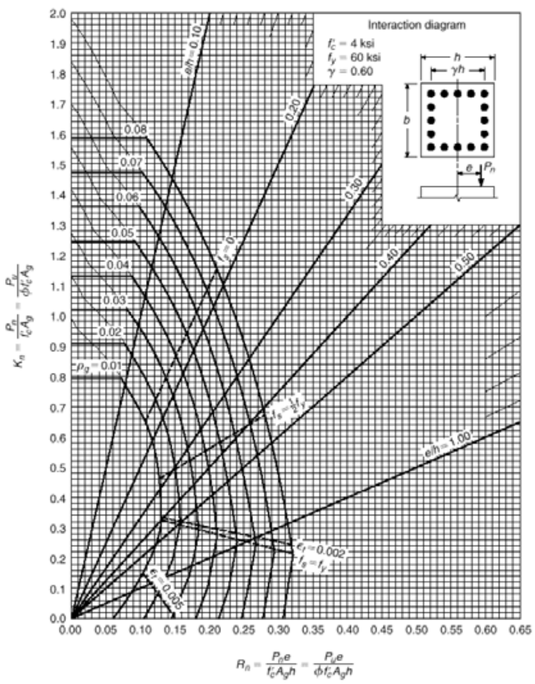
ACI safety provisions superimposed on column strength interaction diagram.

The outer curve represents the actual carrying capacity.
The inner curve shown partially dashed, then solid, then dashed, represents the basic design strength obtained by reducing $\boldsymbol{P_n}$ and $\boldsymbol{M_n}$ to $\boldsymbol{\varphi P_n}$ and $\boldsymbol{\varphi M_n}$.
The horizontal cutoff at $\boldsymbol{\varphi P_o}$ represents the max design load for small $\boldsymbol{e}$.
At the other end (*lower right*), for large $\boldsymbol{e}$, i.e., small axial loads, a linear transition of $\boldsymbol{\varphi}$ from 0.65 or 0.75 to 0.9 at $\boldsymbol{\varepsilon_t} = 0.005$

Design Aids

In practice, design aids are used. They cover the most frequent practical cases for reinforced rectangular and square columns and circular spirally reinforced columns.
Graphs A.5 through A.16 are RC column design charts for concrete with $\boldsymbol{f'_c} = \mathbf{28}$ **MPa** and steel with $\boldsymbol{f_y} = \mathbf{420}$ **MPa:**
Graphs A.5 through A.8: Rectangular columns with bars distributed around perimeter.
Graphs A.9 through A.12: Rectangular columns with bars along 2 opposite faces.
Graphs A.13 through A.16: Circular columns with bars in a circular pattern.

The vertical axis (load) is plotted as $\boldsymbol{K_n = P_u / \varphi f'_c A_g}$
The horizontal axis (moment) is plotted as $\boldsymbol{R_n = P_u e / \varphi f'_c A_g h}$
Families of curves are drawn for various values of $\boldsymbol{\rho_g = A_{st} / A_g}$.
Radial lines represent different eccentricity ratios ($\boldsymbol{e / h}$).
Lines of $\boldsymbol{f_s / f_y}$ and values of strain $\boldsymbol{\varepsilon_t}$ = 0.002 and 0.005 in extreme tension steel are presented.



GRAPH A.5
Column strength interaction diagram for rectangular section with bars on four faces and γ = 0.60 (for instructional use only).

Use of charts:

They may be used in 2 ways as follows.
For a given factored load $\boldsymbol{P_u}$ and equivalent eccentricity $\boldsymbol{e = M_u / P_u}$**:**

1.
   a) Select trial cross section dimensions $\boldsymbol{b}$ and $\boldsymbol{h}$.
   b) Calculate the ratio $\boldsymbol{\gamma}$, and select the corresponding chart.
   c) Calculate $\boldsymbol{K_n = P_u / \varphi f'_c A_g}$ and $\boldsymbol{R_n = P_u e / \varphi f'_c A_g h}$, where $\boldsymbol{A_g = bh}$.
   d) From graph read reinforcement ratio $\boldsymbol{\rho_g}$
   e) Calculate the total steel area $\boldsymbol{A_{st} = \rho_g bh}$.

2.
   a) Select the reinforcement ratio $\boldsymbol{\rho_g}$
   b) Choose a trial value of $\boldsymbol{h}$ and calculate $\boldsymbol{e/h}$ and $\boldsymbol{\gamma}$.
   c) Select the graph, and read $\boldsymbol{K_n = P_u / \varphi f'_c A_g}$ and calculate the required $\boldsymbol{A_g}$.
   d) Calculate $\boldsymbol{b = A_g / h}$**.**
   e) Revise $\boldsymbol{h}$ if necessary to obtain a well-proportioned section.
   f) Calculate the total steel area $\boldsymbol{A_{st} = \rho_g bh}$.

Example 4:

An exterior rectangular column ($\boldsymbol{b}$ **= 500 mm**, $\boldsymbol{h}$ **= 650 mm**) is to be designed for a service dead load of **990 kN**, live load of **1320 kN**, dead load moment of **190 kNm**, and live load moment of **270 kNm**. Find the required column reinforcement. f'_c **= 28 MPa** and f_y **= 420 MPa**.

Solution:

$P_u = 1.2D + 1.6L = 1.2(990) + 1.6(1320) = 3300$ kN

$M_u = 1.2M_D + 1.6M_L = 1.2(190) + 1.6(270) = 660$ kNm

Assume bending is about the strong axis (h = 650).

Reinforcement distributed around column perimeter will be used, and assume $d' = 65$ **mm**

$\gamma = (h - 2d')/h = (650 - 130) / 650 = 0.8$ Thus, **graph A.7** will be used.

$K_n = P_u / \varphi f'_c A_g = 3300 \times 10^3 / (0.65 \times 28 \times 325{,}000) = 0.558$

$R_n = P_u e / \varphi f'_c A_g h = 660 \times 10^6 / (0.65 \times 28 \times 325{,}000 \times 650) = 0.172$

From graph A.7 read $\rho_g = 0.023$

$A_{st} = \rho_g bh = 0.023 \times 325{,}000 = 7475$ mm^2. Use **12No.29** (7740 mm^2), one at each corner and two evenly spaced along each face of column.

Example 5:

A column is designed to carry a factored load P_u **= 2140 kN** and factored moment M_u **= 690 kNm** about the strong axis. Cost studies indicate that ρ_g = **0.03** is optimum with steel arrangement in two layers parallel to the axis of bending. Find the required dimensions b and h of the column. f'_c **= 28 MPa** and f_y **= 420 MPa**.

Solution:

A trial h = 650 mm is selected and assume d' = 65 mm

$\gamma = (h - 2d')/h = (650 - 130) / 650 = 0.8$ Thus, **graph A.11** will be used

$e = M_u / P_u = 690 / 2140 = 0.322$ m = 322 mm

$e / h = 322 / 650 = 0.50$

From graph A.11 with $e / h = 0.50$ and ρ_g **= 0.03, read** K_n **= 0.505**

$K_n = 0.505 = P_u / \varphi f'_c A_g = 2140 \times 10^3 / (0.65 \times 28 \times b \times 650)$,

Find b = 373 mm

Use column **375 × 650 mm** with $A_{st} = \rho_g bh = 0.03 \times 375 \times 650 = 7031$ mm^2.

Use **8No.36** (8048 mm^2), arranged in two layers of **4** bars each.

Biaxial Bending

There are situations in which axial compression is accompanied by simultaneous bending about both principal axes of the section. Such is the case of corner columns of buildings, and of interior columns if the column layout is irregular.

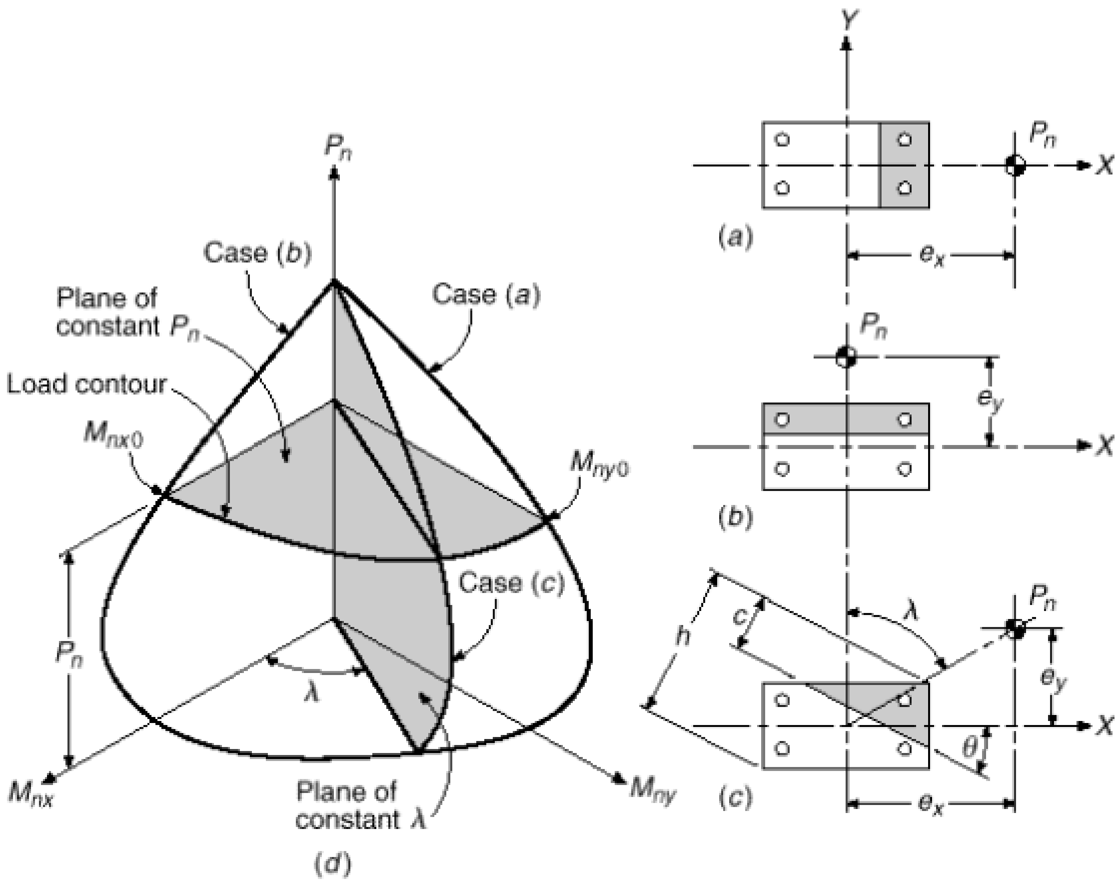
The situation is described in the Figs. below.

Figs a and b describes uniaxial bending about ***Y*** and ***X*** axes, respectively.

Fig. c describes the case of biaxial bending. The orientation of the resultant eccentricity is defined by the angle ***λ*:**

$\lambda = \tan^{-1}(e_x / e_y) = \tan^{-1}(M_{ny} / M_{nx})$

Bending is about an axis defined by the angle *λ* w.r.t. ***X*** axis.



Column strength is defined by the interaction curve labeled case (c). For other values of, similar curves are obtained to define a *failure surface* for axial load plus biaxial bending.

Due to many difficulties related to the subject, a simple approximate method is widely used, as follows.

Reciprocal Load Method

The reciprocal load equation:

$$(1 / P_n) = (1 / P_{nx0}) + (1 / P_{ny0}) - (1 / P_0)$$

Where

P_n = approximate value of nominal load in biaxial bending with eccentricities e_x and e_y

P_{ny0} = nominal load when only e_x is present ($e_y = 0$)

P_{nx0} = nominal load when only e_y is present ($e_x = 0$)

P_0 = nominal load for concentrically loaded column

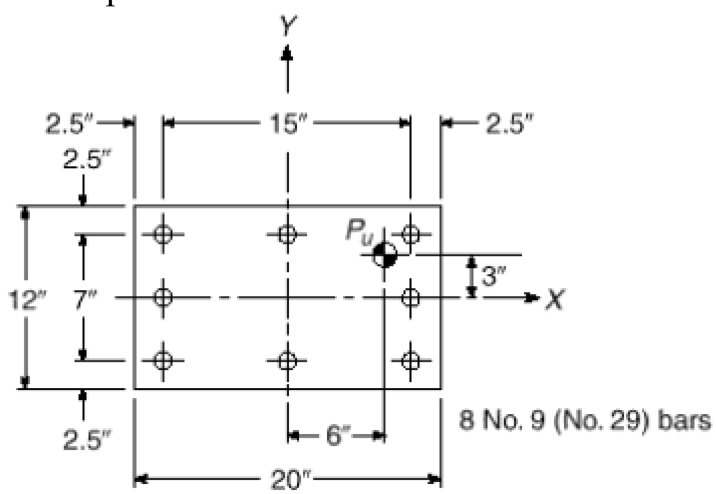
This equation is acceptable provided $P_n \geq 0.10\, P_0$

Example 6:

The **300 × 500 mm** column shown in Fig. is reinforced with **8 No.29** bars arranged around the column perimeter, providing an area A_{st} **= 5160 mm²**

A factored load P_u of **1130 kN** is to be applied with eccentricities e_y **= 3'' (75 mm)** and e_x **= 6'' (150 mm)**. f'_c **= 28 MPa** and f_y **= 420 MPa**.

Check the trial design using the reciprocal load method.



2.5"=65mm, 7"=170mm, 12"=300mm, 15"=370mm

Solution:

1st, bending about **Y** axis:

$\gamma = 370 / 500 = 0.74$, $e / h = 150 / 500 = 0.3$, $A_s / bh = 5160 / (300 \times 500) = 0.0344$

Use weighted average between graph A.6 ($\gamma = 0.7$) and A.7 ($\gamma = 0.8$):

For $\gamma = 0.74$, it means 60% of graph A.6 and 40% of graph A.7:

$P_{ny0} / f'_c A_g$ (ave.) = 0.60 (0.63) + 0.40 (0.66) = 0.64

$P_{ny0} = 0.64 \times 28 \times 150{,}000 \times 10^{-3}$ = **2696 kN**

$P_0 / f'_c A_g = 1.34$, P_0 = **5628 kN** (P_u = 1130 kN > $0.1P_0$ = 562.8 kN OK)

2nd, bending about **X** axis:
γ = 170 / 300 = 0.57, e / h = 75 / 300 = 0.25, A_s / bh = 5160 / (300 × 500) = 0.0344
Use graph A.5 (γ = 0.6):
$P_{nx0} / f'_c A_g$ = 0.67,
P_{nx0} = 0.67 ×28×150,000×10^{-3} = **2814 kN**
$P_0 / f'_c A_g$ = 1.34, P_0 = **5628 kN**
Substitute these values in RLE:
$(1 / P_n) = (1 / P_{nx0}) + (1 / P_{ny0}) - (1 / P_0)$
$(1 / P_n)$ = (1 / 2814) + (1 / 2696) − (1 / 5628) = 0.0005486
P_n = 1823 kN
Thus design load P_u = 0.65×1823 = **1185 kN** can be applied safely.
1130 kN < 1185 kN OK

Bar Splicing in Columns
The main vertical reinforcement in columns is usually spliced above each floor, or sometimes at alternate floors.
It is standard practice to offset the lower bars as shown in Fig. to permit the proper positioning of the upper bars.
According to ACI Code 7.8.1, the slope of the inclined bars of an offset bar **≤ 1 : 6**, and the ties placed **≤ 150 mm (6 in.)** as shown in Fig.
According to ACI Code 7.10.5, ties may be terminated **≤ 75 mm (3 in.)** below the lowest reinforcement of beams framing into a joint, as shown in Fig.