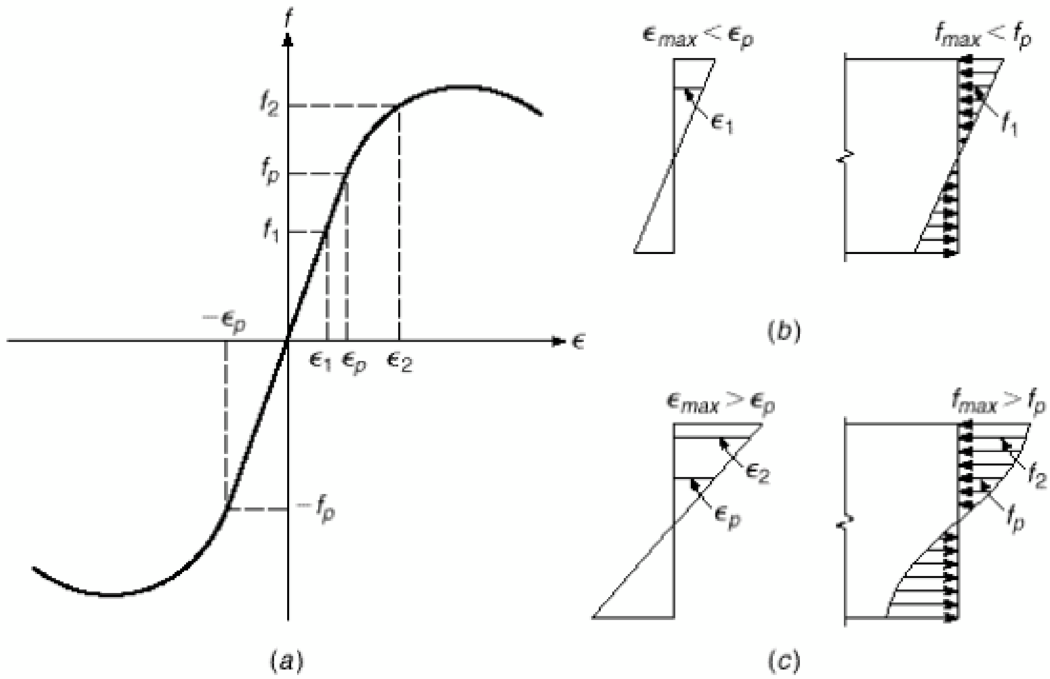
2 Flexural Analysis and Design of RC Beams

Fundamental assumptions

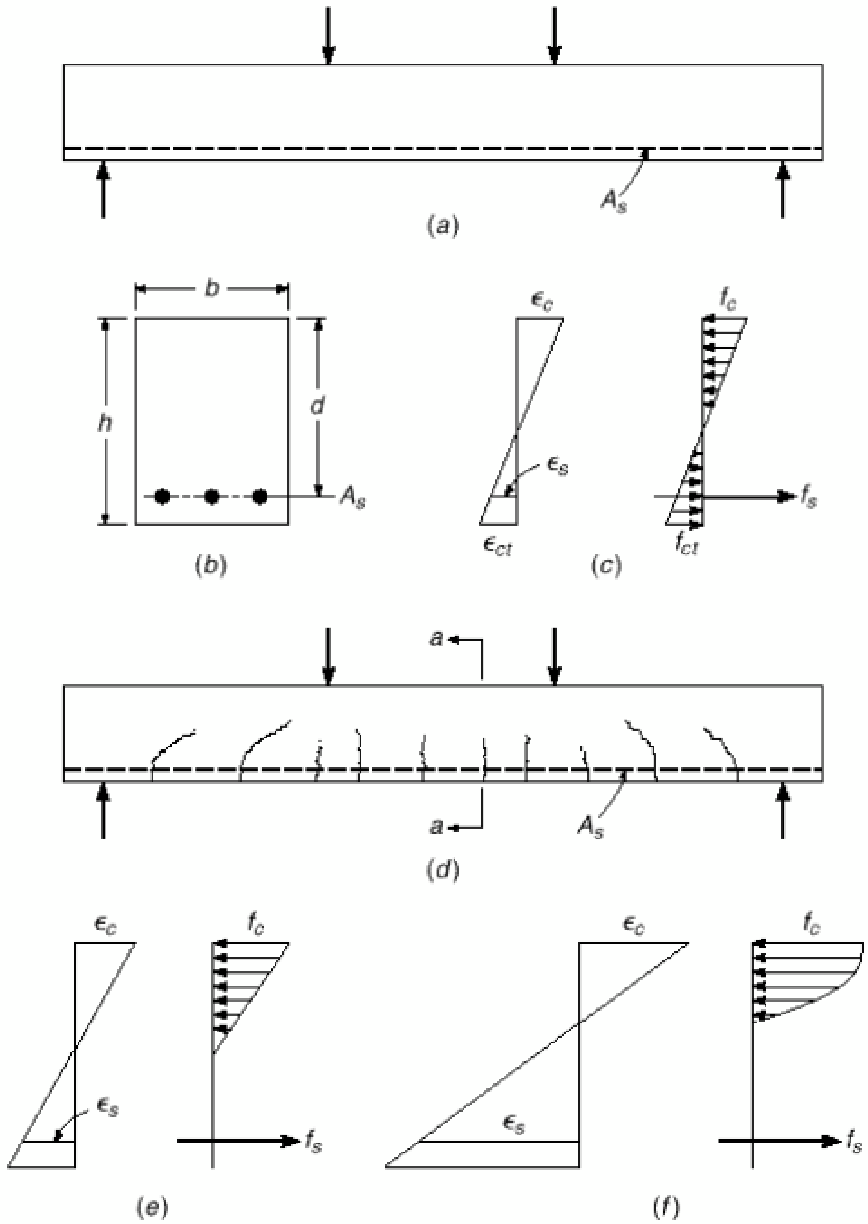
1. A cross section that was plane before loading remains plane under load.
2. The bending stress ***f*** at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material. (See Fig.)



3. Distribution of shear stresses over the depth of the section depends on the shape of the cross section and the stress-strain diagram.
4. At any point in the beam there are inclined stresses of tension and compression, forming an angle of 45° with the horizontal. The largest of which form angle 90° with each other.
5. When the stresses in the outer fibers are smaller than the proportional limit f_p, the beam behaves elastically, as shown in fig. b. In this case the following pertains:

a) The NA passes thru the c.g. of the cross section.
b) The intensity of bending stress increases directly with the distance from NA: $f = M.y / I$, $f_{max} = M.c / I = M / S$, ($S = I / c$)
c) The shear stress at any point is given by $v = V.Q / (I.b)$
d) The intensity of shear stress varies as a parabola being zero at outer fibers and max at NA.

<u>RC Beam Behavior</u>



1st stage (fig. c): At low loads, all stresses are of small magnitude and are proportional to strains.

2nd stage (fig. e): When the load is increased, the tensile strength of concrete is reached; tension cracks develop; concrete does not transmit any tensile stresses. The steel resists the entire tension. If concrete compressive stresses do not exceed ≈ 0.5 f'_c, stresses and strains continue to be proportional (linear stress distribution)

3rd stage (fig. f): When the load is further increased, stresses and strains are no longer proportional; the distribution of concrete stresses on the compression side is of the same shape as concrete stress-strain curve.

Failure can be caused in one of two ways:

A) When moderate amounts of reinforcement are employed, the steel will reach its yield point; the reinforcements stretches a large amount; the tension cracks widen and propagate upward; significant deflection of the beam;
When this happens, the strains in compression zone of concrete increase to ensue crushing (*secondary compression failure*) at a load only slightly greater than that which cause the steel to yield.
Such **yield failure** is gradual and is preceded by visible signs of distress: cracks and deflection.

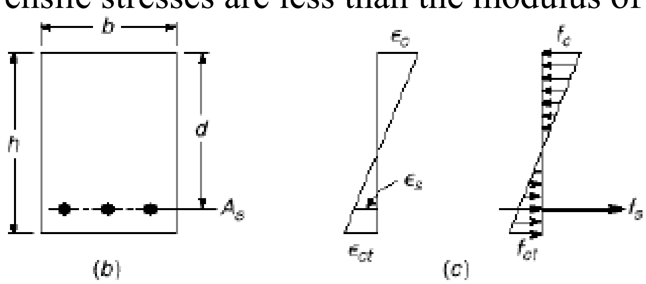
B) **When large amounts of reinforcement are employed,** the compressive strength of concrete is exhausted before the steel starts yielding. It has been observed that beams fail in compression when the concrete strains reach values of about 0.003 to 0.004.
Such **compression failure** is sudden; explosive, and occurs without warning.

It is good practice to dimension beams that they will fail by yielding of the steel (A) rather than by crushing of concrete (B).

Analysis of Stresses and Strength in the Different Stages

a) Stresses Elastic and Section Uncracked (figure c)

Tensile stresses are less than the modulus of rupture f_r.

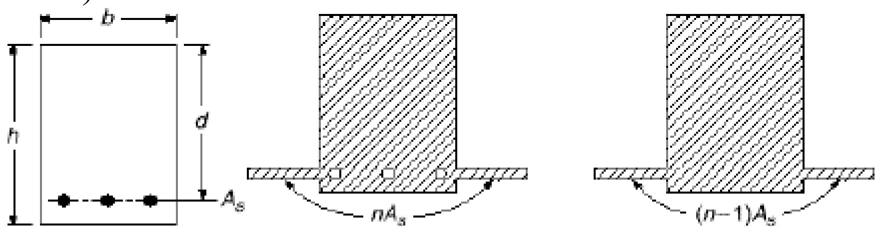


At the level of reinforcement: $\varepsilon_s = \varepsilon_c$;

$$f_s / E_s = f_c / E_c \;;$$
$$f_s = (E_s / E_c)\, f_c$$
$$f_s = n\, f_c$$

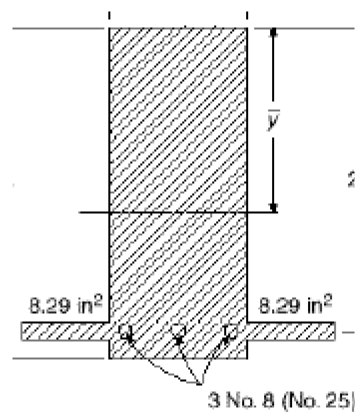
Where $n = E_s / E_c$ is known as the *modular ratio*.

It means that the stress in steel (***$f_s = n\ f_c$***) is ***n*** times that of the concrete. The analysis shall depend on the "transformed section". In this fictitious section, the actual area of the reinforcement is replaced with an equivalent concrete area equal to ***nA_s***, located at the level of steel. (Figure below)



Example 1

A rectangular beam has the dimensions b = 250 mm, h = 650 mm, and d = 600 mm and is reinforced with 3 No. 25 bars so that $A_s = 1530\ mm^2$. The concrete cylinder strength f'_c is 28 MPa, and the tensile strength in bending (modulus of rupture) f_r is 3.27 MPa. The yield point of the steel f_y is 420 MPa. Determine the stresses caused by a bending moment M = 61 kN-m.



Solution

$E_c = 4700 \sqrt{f'_c} = 24870$ MPa

$n = E_s / E_c = 200000 / 24870 = 8$

Add an area $(n - 1)A_s = 7 \times 1530$

$= 10710\ mm^2$ {$5355\ mm^2$ ($8.29\ in^2$) as shown}

$\bar{y} = \sum A.y / \sum A = 342$ mm from top (check this)

$I = 6{,}481{,}000{,}000\ mm^4$ (check this)

Compression stress at top $f_c = M.\bar{y} / I$

$= 61{,}000{,}000 \times 342 / 6{,}481{,}000{,}000 =$ 3.22 MPa

Tension stress at bottom f_{ct} = 61,000,000 × 308 / 6,481,000,000 = 2.90 MPa

Since 2.90 MPa < f_r = 3.27 MPa (given), no tensile cracks will form, and calculation by uncracked transformed section is justified.

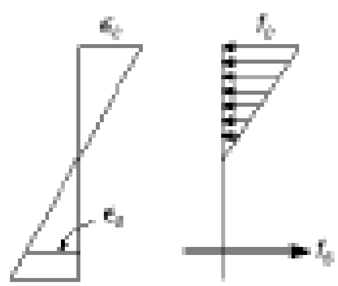
Steel stress $f_s = n\ M.y / I$

$= 8\ (61{,}000{,}000 \times 258 / 6{,}481{,}000{,}000) =$ 19.43 MPa

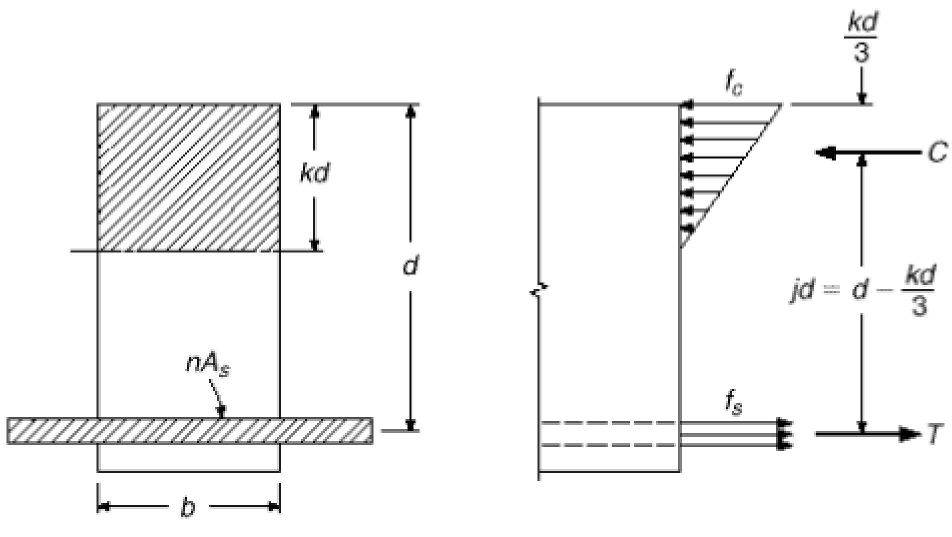
It is seen that at this stage the actual stresses are quite small compared with the available strengths of steel and concrete.

b) Stresses Elastic and Section Cracked (figure e)

When the tensile stress f_{ct} exceeds the modulus of rupture f_r, cracks form. If the compressive stress f_c is less than ≈ 0.5 f'_c and the steel stress has not reached the yield point ($f_s < f_y$), both materials continue to behave elastically.
This situation occurs in structures under normal service conditions and loads. This situation with regard to strain and stress distribution is that shown in figure e:



The fact is that all of the concrete that is stressed in tension is assumed cracked, and therefore effectively absent. (Figure below: cracked transformed section)



To determine the location of the N.A. (*kd* from top), the moment of the tension area about the axis is set equal to the moment of the compression area:

$$b(kd)^2/2 - nA_s(d - kd) = 0 \qquad \text{.......(1)}$$

Then determine the moment of inertia.

Alternatively,

$$C = \frac{f_c}{2} bkd \qquad \text{and} \qquad T = A_s f_s \tag{3.6}$$

The requirement that these two forces be equal numerically has been taken care of by the manner in which the location of the neutral axis has been determined.

Equilibrium requires that the couple constituted by the two forces C and T be equal numerically to the external bending moment M. Hence, taking moments about C gives

$$M = Tjd = A_s f_s jd \tag{3.7}$$

where jd is the internal lever arm between C and T. From Eq. (3.7), the steel stress is

$$f_s = \frac{M}{A_s jd} \tag{3.8}$$

Conversely, taking moments about T gives

$$M = Cjd = \frac{f_c}{2} bkdjd = \frac{f_c}{2} kjbd^2 \tag{3.9}$$

from which the concrete stress is

$$f_c = \frac{2M}{kjbd^2} \tag{3.10}$$

In using Eqs. (3.6) through (3.10), it is convenient to have equations by which k and j may be found directly, to establish the neutral axis distance kd and the internal lever arm jd. First defining the *reinforcement ratio*

$$\boldsymbol{\rho = A_s / bd}$$

then substituting $\boldsymbol{A_s = \rho\, bd}$ into equation (1), solving for $\boldsymbol{k}$

$$\boldsymbol{k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n}$$

then $\boldsymbol{jd = d - kd/3,}$

$$\boldsymbol{j = 1 - k/3}$$

Values of $\boldsymbol{k}$ and $\boldsymbol{j}$ are tabulated (Nilson Table A6):

Example 2

The beam of Example 1 is subjected to a bending moment M = 122 kN-m (rather than 61 kN-m). Calculate the relevant properties and stresses.

Solution

Check the section is cracked:
Tension stress at bottom f_{ct} = 122,000,000 × 308 / 6,481,000,000 = 5.8 MPa

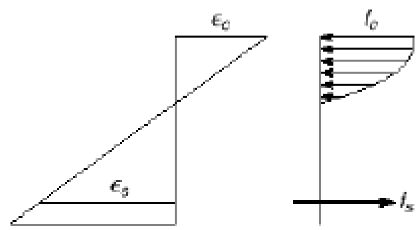
Since 5.8 MPa > f_r = 3.27 MPa (given), tensile cracks will form, and calculation must adapt the cracked transformed section.
Equation 1, $b(kd)^2/2 - nA_s(d - kd) = 0$, with b = 250 mm, d = 600 mm, n = 8, and A_s = 1530 mm^2 inserted, gives
kd = 198 mm (distance to N.A.)
k = 198/600 = 0.33,
$j = 1 - k/3 = 0.89$
$f_s = M / A_s jd$ = 122,000,000/[1530 × 0.89 × 600] = <u>149.3 MPa</u>
$f_c = 2M / kjbd^2$ = 2×122,000,000/[0.33×0.89×250×600^2] = <u>9.23 MPa</u>
or
$\boldsymbol{\rho = A_s / bd}$ = 1530 / (250 × 600) = 0.0102, $\boldsymbol{\rho n}$ = 0.0102× 8 = 0.0816

$\boldsymbol{k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n} = \sqrt{(0.0816)^2 + 2(0.0816)} - 0.0816 = 0.33$ as before.

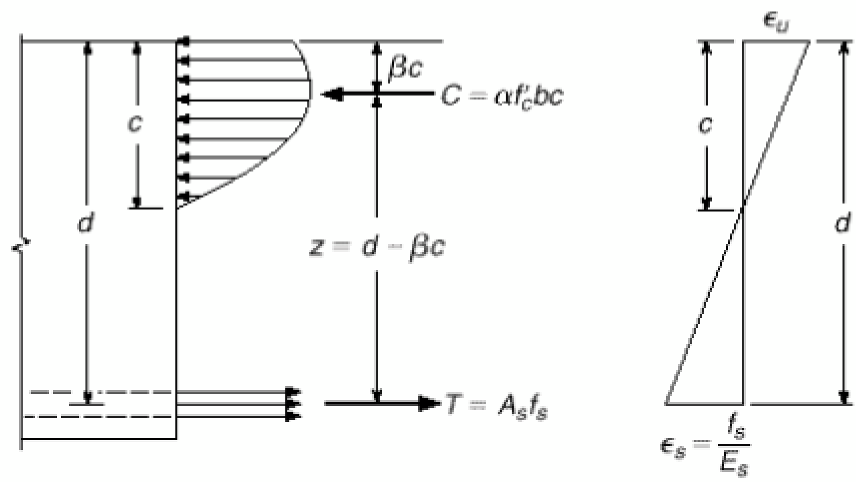
Notes: (compared with Example 1; doubling M)
1. N.A. has moved upward: changed from 342 to 198 mm.
2. The steel stress changed from 19.43 to 149.3 MPa (about 8 times).
3. The concrete compressive stress has increased from 3.22 to 9.23 MPa (about 3 times).
4. The moment of inertia of cracked section (2,625,000,000 mm^4 check this!) is less than that of uncracked section (6,481,000,000 mm^4). This affects the magnitude of the deflection.

c) Flexural Strength (figure f)

At high loads, close to failure, the distribution of stresses and strains is that of fig. f:



Stress and strain distributions at ultimate load are assumed as shown in fig. below:



For failure mode A, two criteria are implied

- $f_s = f_y$
- The concrete crushes when the maximum strain reaches ε_u **= 0.003**.

It is necessary to know, for a given distance c of N.A.,
1. The total resultant compression force C in the concrete.
2. Its vertical location, i.e., its distance from the outer compression fiber.

In rectangular beams, area in compression is bc, and $C = f_{av}bc$

Let $\alpha = f_{av} / f'_c$ then $C = \alpha f'_c bc$
The location of C is at βc from top.

Knowing **α** and **β** will define the compressive stresses.

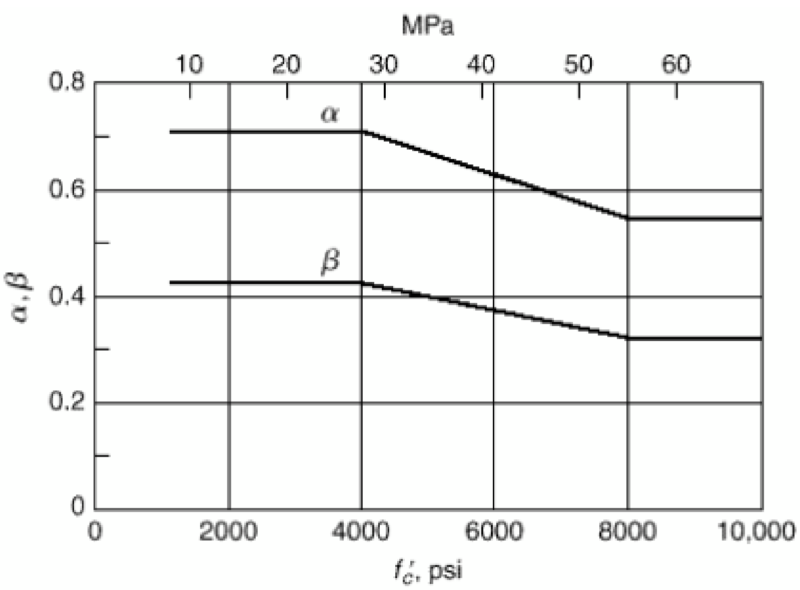
If **α** and **β** are known, then equilibrium requires that
$C = T$ or $\alpha f'_c bc = A_s f_s$
Also
$M = Tz = A_s f_s (d - \beta c)$
Or $M = \alpha f'_c bc(d - \beta c)$

Set $f_s = f_y$ then $c = A_s f_y / \alpha f'_c$
Using $A_s = \rho bd$, then $c = \rho f_y d / \alpha f'_c$

Substitute, M_n is then obtained

$$M_n = \rho f_y bd^2(1 - \beta \rho f_y / \alpha f'_c)$$

From extensive experimental work, the values of α and β have shown to be as in the figure below (for $f'_c \le 28$ MPa, α = 0.72 and β = 0.425)



Now, the nominal moment equation becomes:

$$M_n = \rho f_y bd^2(1 - 0.59 \rho f_y / f'_c)$$

Balanced reinforcement ratio ρ_b

The balanced reinforcement ratio, ρ_b represents that amount of reinforcement necessary for the beam to fail by crushing of the concrete at the same load that causes the steel to yield.

Hooke's law: $f_s = \varepsilon_s E_s$

From strain distribution (see fig.), similar triangles give

$$f_s = \varepsilon_u E_s(d - c)/c$$

Setting $f_s = f_y$, and substituting ε_y for f_y/E_s, the value of c defining the unique position of the N.A. corresponding to simultaneous crushing of the concrete and initiation of yielding in the steel,

$$c = d.\ \varepsilon_u/(\varepsilon_u + \varepsilon_y)$$

Substituting c in equation $C = T$ or $\alpha f'_c\, bc = A_s f_s$ with $A_s f_s = \rho b d f_y$, the ρ_b is obtained

$$\rho_b = (\alpha f'_c / f_y)[\ \varepsilon_u/(\varepsilon_u + \varepsilon_y)]$$

Example 3

Determine the nominal moment M_n at which the beam of Examples 1 and 2 will fail.

Solution

$\rho = A_s / bd = 1530 / (250\times600) = 0.0102$ (*always write ρ with 4 digits*)

check

$\rho_b = (\alpha f'_c / f_y)[\ \varepsilon_u/(\varepsilon_u + \varepsilon_y)] = 0.0282$ $(\alpha = 0.72)$

Since $\rho < \rho_b$, the beam will fail in tension by yielding of the steel, its nominal moment is

$$M_n = \rho f_y bd^2(1 - 0.59\,\rho f_y/f'_c)$$
$$= 0.0102 \times 420 \times 250 \times 600^2 (1 - 0.59 \times 0.0102 \times 420 / 28)$$
$$= 350{,}800{,}000 \text{ N-mm} = 350.8 \text{ kN-m}$$

At this M_n, the distance to neutral axis is

$c = \rho f_y d / \alpha f'_c$

$= 0.0102 \times 420 \times 600 / (0.72 \times 28) = 127.5$ mm

Summary

	Ex 1:Uncracked	**Ex 2:Cracked**	**Ex 3:Ultimate**
NA from top,mm	342	198	127.5
f_c/f_s (MPa/MPa)	3.22 / 19.43	9.23 / 149.3	28 / 420
M, kN-m	61	122	350.8

The differences between various stages (as the load is increased) are

1. The migration of the N.A. toward the compression edge.
2. The increase in steel stress.
3. The increase in concrete compressive stress.

Design of Tension-Reinforced Rectangular Beams

To provide sufficient strength to RC structures:

1. The nominal strength is modified by a ***strength reduction factor*** **φ**, less than unity, to obtain the *design strength*.
2. The *required strength* is found by applying ***load factors*** **γ**, greater than unity, to loads actually expected (*service loads*).

Thus, RC members are proportioned such that $M_u \le \varphi M_n$; $V_u \le \varphi V_n$; $P_u \le \varphi P_n$

where subscripts ***n*** denote the nominal strengths in flexure, shear, and axial load respectively, and ***u*** denote the factored load moment, shear and axial load.

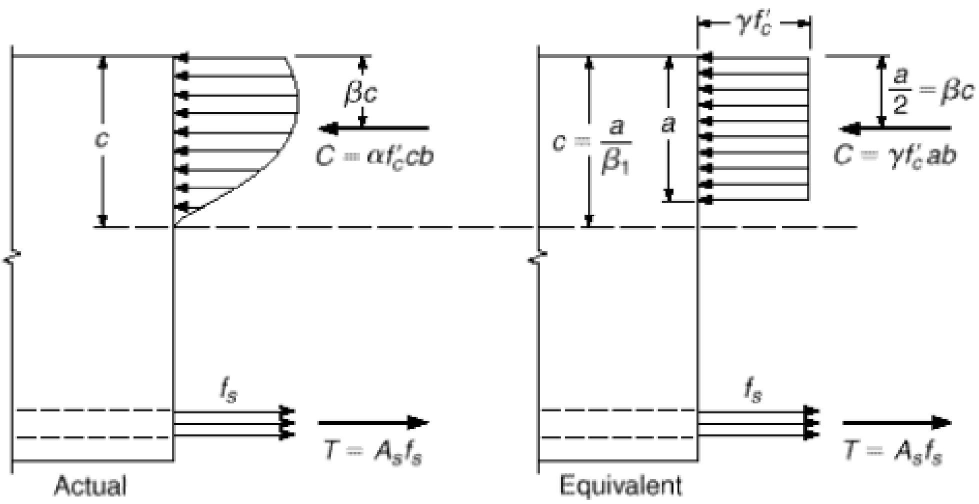
See page 4, chapter 1 of the lecture notes.

a. Equivalent Rectangular Stress Distribution

It was noted that the actual shape of the concrete compressive stress distribution varies considerably. The magnitude ***C*** and location ***βc*** of the resultant of the concrete compressive stresses are obtained from experiments and expressed in the two parameters ***α*** and ***β***.

For simplicity, the actual stress distribution is replaced by an equivalent one of simple rectangular outline. See next figure.

The conditions are that the magnitude of ***C*** and its location must be the same in the equivalent rectangular as in the actual stress distribution.



(a) (b)

$C = \alpha f'_c bc = \gamma f'_c ab$ from which $\gamma = \alpha c / a$

With $a = \beta_1 c$, this gives $\gamma = \alpha / \beta_1$

The force C is located at the same distance: $\beta_1 = 2\beta$.

$\gamma = \alpha / \beta_1 = \alpha / 2\beta$ is seen independent of f'_c and can be taken as 0.85 throughout (e.g. 0.72/(2×0.425) = 0.85):

The force ***C***: $C = 0.85\, f'_c\, ab$

The distance ***a***: $a = \beta_1 c$

$\beta_1 = 0.85$ …………………… for $f'_c \le 28$ **MPa**

$\beta_1 = 0.85 - 0.05\,(f'_c - 28)\,/\,7$ ………….. for $f'_c > 28$ **MPa**

$$0.65 \le \beta_1 \le 0.85$$

b. Balanced Strain Condition

From strain diagram $c = d.\ \varepsilon_u / (\varepsilon_u + \varepsilon_y)$

Equilibrium C = T ; $0.85\, \beta_1 f'_c\, bc = \rho_b bdf_y$

$$\rho_b = 0.85\, \beta_1\, (f'_c / f_y)\,[\, \varepsilon_u / (\varepsilon_u + \varepsilon_y)]$$

c. Under-reinforced Beams

To ensure that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete, this can be done, theoretically by requiring

$$\rho < \rho_b$$

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons:

1. To get significant yielding before failure.
2. Material properties are never known exactly.
3. Strain-hardening of the steel may lead to concrete compression failure.
4. Actual steel area provided will always be equal to or larger than required.
5. Lower ρ increases deflection and thus provides warning prior to failure.

d. ACI Code Provisions for Under-reinforced Beams

ACI Code defines the safe limits of maximum reinforcement by two forms both are based on the ***net tensile strain*** ε_t of the reinforcement farthest from the compression face of the concrete at depth d_t

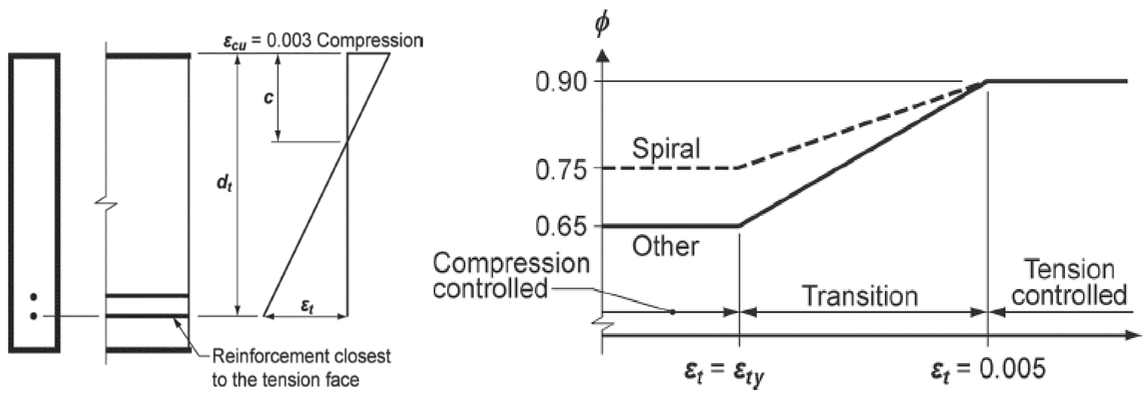


Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

Net tensile strain ε_t	Classification	ϕ — Type of transverse reinforcement: Spirals conforming to 25.7.3		Other	
$\varepsilon_t \le \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < 0.005$	Transition[1]	$0.75+0.15\dfrac{(\varepsilon_t-\varepsilon_{ty})}{(0.005-\varepsilon_{ty})}$	(c)	$0.65+0.25\dfrac{(\varepsilon_t-\varepsilon_{ty})}{(0.005-\varepsilon_{ty})}$	(d)
$\varepsilon_t \ge 0.005$	Tension-controlled	0.90	(e)	0.90	(f)

[1]For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

1. The minimum tensile reinforcement strain allowed at nominal strength:

$\boldsymbol{\varepsilon_t = \varepsilon_u (d_t - c) / c}$

$\boldsymbol{\rho = 0.85\, \beta_1 (f'_c / f_y)(d_t / d)[\varepsilon_u / (\varepsilon_u + \varepsilon_t)]}$

conservatively

$\boldsymbol{\rho = 0.85\, \beta_1 (f'_c / f_y)[\varepsilon_u / (\varepsilon_u + \varepsilon_t)]}$

To ensure under-reinforced behavior, ACI Code 21.2 establishes a minimum net tensile strain ε_t at nominal of 0.004 for members subjected to axial loads less that $\mathbf{0.10}\, f'_c A_g$, where is the gross area of the cross section. Substituting in ρ equation:

$$\boxed{\rho = 0.85\, \beta_1 (f'_c / f_y)[\varepsilon_u / (\varepsilon_u + 0.004)]}$$

2. Allowing ***strength reduction factors*** that depend on the tensile strain at nominal strength. The Code defines:

a. ***Tension-controlled member***: The one with a net tensile strain $\varepsilon_t \ge \mathbf{0.005}$. the corresponding strength reduction factor $\varphi = \mathbf{0.9}$.

b. ***Compression-controlled member:*** The one with a net tensile strain $\boldsymbol{\varepsilon_t \le \varepsilon_{ty} = 0.002}$. The corresponding strength reduction factor $\boldsymbol{\varphi = 0.65}$. For spirally-reinforced members $\boldsymbol{\varphi = 0.75}$

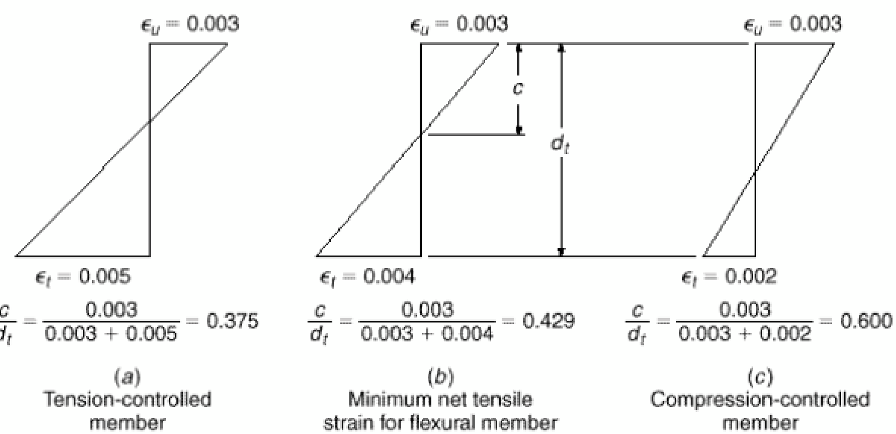
For ε_t between 0.002 and 0.005, φ varies linearly, and ACI Code allows linear interpolation of φ based on ε_t. See Figure and Table 21.2.2 above

The maximum reinforcement ration for a tension-controlled beam is: (recommended for flexural members)

$$\rho_{0.005} = 0.85\, \beta_1 \,(f'_c / f_y)[\varepsilon_u /(\varepsilon_u + 0.005)]$$

The depth of equivalent rectangular stress block a:
Since $c = a / \beta_1$, it is more convenient to compute c/d_t rather than ρ or net ε_t, see Figure. Maximum value of $\boldsymbol{c/d_t = 0.375}$ for $\boldsymbol{\varepsilon_t \ge 0.005}$

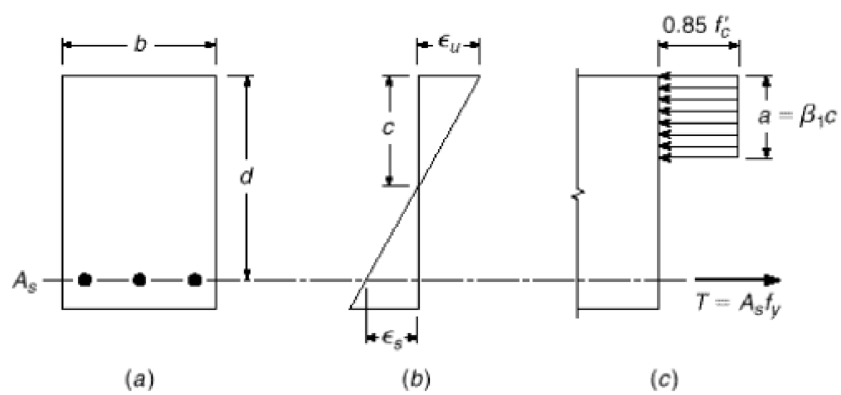
FIGURE 3.10
Net tensile strain and c/d_t ratios.



The nominal flexural strength is given by (see figure below)

$$M_n = A_s f_y (d - a/2), \quad a = A_s f_y / 0.85\, f'_c\, b$$

FIGURE 3.11
Singly reinforced rectangular beam.



Example 4

Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 3. Recall $\boldsymbol{b}$ = 250 mm, $\boldsymbol{d}$ = 600 mm, $\boldsymbol{A_s}$ = 1530 mm2, $\boldsymbol{f'_c}$ = 28 MPa, $\boldsymbol{f_y}$ = 420 MPa.

Solution

$\beta_1 = 0.85$ ($f'_c = 28$ MPa)

$\rho_{0.005} = 0.85\,\beta_1 (f'_c / f_y)[\varepsilon_u / (\varepsilon_u + 0.005)]$

$= 0.85 \times 0.85\ (28/420)[0.003 / (0.003 + 0.005)] = 0.0181$

Actual $\rho = 1530 / (250 \times 600) = 0.0102$

Since $\rho < \rho_{0.005}$, the member will fail by yielding of steel.

Alternatively, recall c = 127.5 mm,

$c/d_t = 127.5/600 = 0.213 < 0.375$, the member will fail by yielding of steel

$a = A_s f_y / 0.85 f'_c\, b = 1530 \times 420 / (0.85 \times 28 \times 250) = 108$ mm

$M_n = A_s f_y (d - a/2) = 1530 \times 420 (600 - 108/2) = 350.9 \times 10^6$ Nmm = <u>350.9kNm</u>

Moment equation can be re written (as derived previously) as follows:

$$M_n = \rho\, f_y\, bd^2 (1 - 0.59\, \rho\, f_y / f'_c)$$

$= 0.0102 \times 420 \times 250 \times 600^2 (1 - 0.59 \times 0.0102 \times 420/28)[10^{-6}] =$ <u>350.8 kNm</u>

This equation may be simplified further for everyday design as follows

$$M_n = Rbd^2$$

In which

$$R = \rho\, f_y\, (1 - 0.59\, \rho\, f_y / f'_c) \qquad \text{(MPa)}$$

The values of the *flexural resistance factor* $\boldsymbol{R}$ are tabulated in Appendix A5 (Nilson)

In accordance with safety the safety provisions of the ACI Code, the *nominal flexural strength* M_n is reduced by imposing the strength reduction factor φ to obtain the *design strength* φM_n

$$\varphi M_n = \varphi A_s f_y (d - a/2)$$

Or, alternatively, $$\varphi M_n = \varphi \rho f_y bd^2 (1 - 0.59 \rho f_y / f'_c)$$

Or $$\varphi M_n = \varphi R bd^2$$

Example 4 (*continued*): Since $\rho < \rho_{0.005}$ (or $c/d_t < 0.375$), then $\varepsilon_t > 0.005$. Therefore, $\varphi = 0.9$ and design capacity is $\varphi M_n = 0.9 \times 350.9 = 315.8$ kNm

e. Minimum Reinforcement Ratio

In very lightly reinforced beams, if the flexural strength < the moment that produce cracking, the beam will fail immediately and without warning upon formation of the first flexural crack.

To ensure against this type of failure, a ***lower limit*** can be established for the reinforcement.

According to ACI Code 9.6, at any section where tensile reinforcement is required by analysis, the area A_s provided must not be less than

$$A_{s,min} = \rho_{min} b_w d$$

$$\rho_{min} = 0.25 \sqrt{f'_c} / f_y \geq 1.4 / f_y$$

f. Examples of Rectangular Beams

Example 5 (*Analysis problem*)

A rectangular beam has width **300 mm** and effective depth **440 mm**. it is reinforced with **4 No.29** (#9) bars in one row. If f_y = **420 MPa** and f'_c = **28 MPa**, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to ACI Code?

Solution:

Area of 4 No.29 bars = 4 × 645 = 2580 mm^2 (Table A2, Nilson)

$a = A_s f_y / 0.85 f'_c b$ = 2580×420/(0.85×28×300) = 151.8 mm

$c = a / \beta_1$ = 151.8 / 0.85 = 178.5 mm

c / d_t = 178.5 / 440 = 0.406 (between 0.429 and 0.375)

i.e. (ε_t between 0.004 and 0.005)

Thus, the beam is under-reinforced

or, $\rho = A_s/bd$ = 2580 / (300 × 440) = <u>0.0195</u> which just exceeds

$\rho_{0.005} = 0.85\,\beta_1\,(f'_c/f_y)[\varepsilon_u/(\varepsilon_u + 0.005)]$

$= 0.85 \times 0.85\,(28/420)[0.003/(0.003 + 0.005)] = 0.0181$

Since $\varepsilon_t = \varepsilon_u(d - c)/c = 0.003(440 - 178.5)/178.5 = 0.00439$

Using interpolation $\varphi = 0.85$ (*Table 21.2.2*, *show the interpolation in your answer*)

$\varphi M_n = \varphi A_s f_y(d - a/2) = 0.85 \times 2580 \times 420(440 - 151.8/2)$

$= 0.85 \times 394.5 \times 10^6$ Nmm = <u>335.3 kNm</u>

<u>Check</u> $\rho_{max} = \rho_{0.004} = 0.0206$, and $\rho_{min} = 0.25\sqrt{28}/420 \geq 1.4/420 = 0.0033$. Thus $\rho_{min} < \rho = 0.0195 < \rho_{max}$ is satisfactory.

<u>**Example 6**</u> (*Design problem*)

Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of **4.5 m** that is to carry a computed dead load of **20 kN/m** and a service live load of **31 kN/m**. Material strengths are f'_c **= 28 MPa** and f_y **= 420 MPa**.

<u>**Solution:**</u>

Factored load $w_u = 1.2\,D + 1.6\,L$

$= 1.2 \times 20 + 1.6 \times 31 = 73.6$ kN/m

$M_u = w_u\,l^2/8 = 73.6 \times 4.5^2/8 = 186.3$ kNm

To minimize section dimensions, it is desirable to select the maximum permissible reinforcement ratio:

$\rho_{0.005} = 0.85\,\beta_1\,(f'_c/f_y)[\varepsilon_u/(\varepsilon_u + 0.005)]$

$= 0.85 \times 0.85\,(28/420)[0.003/(0.003 + 0.005)] = 0.0181$

$\varphi = 0.9$ ($\varepsilon_t = 0.005$)

$M_u = \varphi\,M_n$

$186.3 \times 10^6 = 0.9 \times 0.0181 \times 420\,bd^2\,(1 - 0.59 \times 0.0181 \times 420/28)$

$bd^2 = 32{,}420{,}000$ mm^2

Say <u>b = 250 mm</u>, <u>d = 360 mm</u>, then

A_s, *required* = 0.0181 × 250 × 360 = 1630 mm^2

<u>USE 2 No.32 (1638 mm^2)</u>

Total depth of section $h = d_t + d_b/2 + d_b$ (*stirrup*) **+ concrete cover**

= 360 + 16 + 12 + 40 = 428 mm

Round – up to the nearest 25 mm: <u>h = 450 mm</u>.

<u>Note:</u>

1. *The effective depth will be increased:* $d = 450 - 40 - 12 - 16 = 382$ *mm. Improved economy may be possible by refining the steel area based on the actual, larger* **d**.
2. *Infinite number of solutions is possible depending upon the reinforcement ratio selected.*

Example 7: (*Design problem: section dimensions are given, A_s is required*)
Find the steel area required to resist a moment M_u of **150 kNm** using a concrete section having b = **250 mm**, d = **435 mm**, and h = **500 mm**. f'_c = **28 MPa** and f_y = **420 MPa.**

Solution:

Assume a = 100 mm

$\varphi M_n = \varphi A_s f_y(d - a/2)$

$A_s = \varphi M_n / \varphi f_y(d - a/2) = 150\times10^6 / 0.9\times420\ (435 - 100/2) = 1031\ mm^2$

Check $a = A_s f_y / 0.85 f'_c\ b = 1031\times420 / 0.85\times 28 \times250 = 72.8$ mm

Next assume a = 70 mm and recalculate A_s:

$A_s = 150\times10^6 / 0.9\times420\ (435 - 70/2) = 992\ mm^2$

No further iteration is required. ***USE*** $A_s = 992\ mm^2$ ***(2 No.25 bars*** $A_s = 1080\ mm^2$***)***

Check $\rho = 0.0091 < \rho_{0.005}$ ***, then*** $\varphi = 0.9$ ***OK***

Example 8: (*section dimensions are given, A_s is required with variable φ*)
Architectural considerations limit the height of a **6 m** long simple span beam to **400 mm** and the width to **300 mm**. the following loads and material properties are given: w_d = **11 kN/m,** w_l = **24 kN/m,** f'_c = **35 MPa, and** f_y = **420 MPa**. Determine the reinforcement of the beam.

Solution:

Factored load $w_u = 1.2 \times 11 + 1.6 \times 24 = 51.6$ kN/m

$M_u = w_u\ l^2 / 8 = 51.6 \times 6^2 / 8 = 232.2$ kNm

Assume a = 100 and φ = 0.9

d = 400 – 65 = 335 mm (assuming 65 mm concrete cover from centroid of bars)

$A_s = M_u / \varphi f_y(d - a/2) = 232.2 \times 10^6/[0.9\times420(335 - 50)] = 2156\ mm^2$

Try 2 No.32 bars and 1 No. 29 bar, $A_{s,\ provided} = 2283\ mm^2$

Check $a = A_s f_y / 0.85 f'_c\ b = 2283\times420 / 0.85\times 35 \times300 = 107.4$ mm

107.4 mm > 100 mm assumed; continue

$M_n = A_s f_y(d - a/2) = 2283\times420\ (335 - 107.4/2)\ 10^{-6} = 269.7$ kNm

$M_u = \varphi M_n = 0.9 \times 269.7 = 242.7$ kNm (adequate: > 232.2 applied)

To validate the selection of φ = 0.9, the net ε_t must be checked:

$c = a / \beta_1 = 107.4 / 0.80 = 134.3$ mm.

$c/d = 134.3/335 = 0.401 > 0.375$ so $\varepsilon_t > 0.005$ is not satisfied.

$\varepsilon_t = 0.003\ (335 - 134.3)/ 134.3 = 0.00448$

From ($\varepsilon_t - \varphi$)Table 21.2.2.; $\varphi = 0.857$

$M_u = \varphi M_n = 231$ kNm (not good: < 232.2)

<u>Try increasing the reinforcement to 3 No.32 bars</u>; $A_{s,\ provided} = 2457\ mm^2$.

Repeating the calculations:
$\boldsymbol{a = A_s f_y / 0.85 f'_c b} = 2457 \times 420 / 0.85 \times 35 \times 300 = 115.6$ mm
$\boldsymbol{c} = \boldsymbol{a / \beta_1} = 115.6 / 0.80 = 144.5$ mm
$\boldsymbol{M_n = A_s f_y (d - a/2)} = 2457 \times 420\ (335 - 115.6/2)\ 10^{-6} = 286.1$ kNm
$\boldsymbol{\varepsilon_t} = 0.003\ (335 - 144.5)/ 144.5 = 0.00400$
$\boldsymbol{\varphi} = 0.65 + 0.25\ (0.00400 - 0.002) /(0.003) = 0.817$
$\boldsymbol{M_u = \varphi M_n} = 0.817 \times 286.1 = 233.7$ kNm (adequate > 232.2)

g. <u>Over-reinforced Beams</u>

Occasionally, it may be necessary to calculate the flexural strength of an over-reinforced compression controlled member for which $\boldsymbol{f_s < f_y}$ at flexural failure.
In this case, $\boldsymbol{\varepsilon_s} < \boldsymbol{\varepsilon_y}$; in terms of $\boldsymbol{\varepsilon_u}$ and $\boldsymbol{c}$:
$\boldsymbol{\varepsilon_s = \varepsilon_u (d - c) / c}$
Equilibrium; $\boldsymbol{0.85\ \beta_1 f'_c\ bc = \varepsilon_s E_s\ bd}$
Substituting and defining $\boldsymbol{k_u = c/d}$:
$\boldsymbol{k_u^2 + m\ \rho k_u - m\rho = 0}$
where $\boldsymbol{\rho = A_s / bd}$ and $\boldsymbol{m = E_s\ \varepsilon_u / (0.85\ \beta_1 f'_c)}$

Solving for $\boldsymbol{k_u}$:

$$\boldsymbol{k_u = [\ m\rho + (\ m\rho/2)^2]^{1/2} - m\rho/2}$$

Then: $\boldsymbol{c = k_u d}$, $\boldsymbol{a = \beta_1 c}$ and $\boldsymbol{\varepsilon_s}$ is known from equation of equilibrium:

$$\boldsymbol{f_s = \varepsilon_s E_s}$$

$$\boldsymbol{\varphi M_n = \varphi A_s f_s (d - a/2) \qquad \varphi = 0.65}$$

h. <u>Design Aids</u>

In practice, the design of beams and other RC members is greatly facilitated by the use of *design aids*.
Tables A.1, A.2, A.4 through A.7, and Graph A.1 relate directly to this chapter.
For design purposes, there are two possible approaches:

1[st] Approach: Start with selecting the optimum ρ and then calculating concrete dimensions, as follows:

1. Set $M_u = \varphi R b d^2$
2. **Table A.4:** Select an appropriate ρ between ρ_{max} and ρ_{min} . Often a ratio of about (0.6 ρ_{max}) will be an economical and practical choice.
   If $\rho \le \rho_{0.005}$ then $\varphi = 0.9$
   If $\rho_{0.005} < \rho < \rho_{max}$ then an iterative solution is necessary.
3. **Table A.5:** Find the flexural resistance factor R. Then $bd^2 = M_u / \varphi R$
4. Choose b and d to meet that requirement. Often d = 2 to 3 times b is appropriate.
5. Calculate $A_s = \rho\ bd$, then use **Table A.2** to choose the size and no. of bars.
6. Refer to **Table A.7** to ensure that the selected beam width will provide room for the bars chosen, with adequate concrete cover and spacing.

2[nd] Approach: Start with selecting concrete dimensions, after which the required reinforcement is found, as follows:

1. Select b and d, then calculate $R = M_u / \varphi b d^2$
2. Use **Table A.5** to find $\rho < \rho_{max}$
3. Calculate $A_s = \rho\ bd$ then use **Table A.2** to choose the size and no. of bars.
4. Refer to **Table A.7** to ensure that the selected beam width will provide room for the bars chosen, with adequate concrete cover and spacing.

Example 9: *USE DESIGN AIDS*

A rectangular beam has width **300 mm** and effective depth **440 mm**. it is reinforced with **4 No.29** (#9) bars in one row. If f_y **= 420 MPa** and f'_c **= 28 MPa**, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to ACI Code?

Solution:

From Table A.2, 4 No.29 bars provide $A_s = 2580\ mm^2$

$\rho = A_s / bd = 0.0195$

Table A.4: this ρ is below ρ_{max} (0.0206) and above ρ_{min} (0.0033)

Table A.5b: $R = 6.79$ MPa

$M_u = \varphi M_n = 0.857\ R b d^2 =$ 335 kNm

Example 10: *USE DESIGN AIDS*
Find the concrete cross section and the steel area required for a simply supported rectangular beam. $\boldsymbol{M_u}$ = 186.3 kNm. Material strengths are $\boldsymbol{f'_c}$ **= 28 MPa** and $\boldsymbol{f_y}$ **= 420 MPa**.

Solution:

Table A.4, $\rho_{max} = 0.0206$. For economy $\rho = 0.6(0.0206) = 0.0124$.
Table A.5a, by interpolation $\boldsymbol{R}$ = 4.63 MPa. Then
$bd^2 = M_u / \varphi R = 44.71 \times 10^6$ mm^3
$\boldsymbol{b}$ = 250 mm and $\boldsymbol{d}$ = 422.9 mm will satisfy this, but the depth will be rounded to 435 mm, to provide a total depth of 500 mm. It follows that
$R = M_u / \varphi bd^2 = 4.83$ MPa
Table A.5a, by interpolation, $\rho = 0.0116$
$A_s = 0.0112\ (250)(435) = 1262$ mm^2

Example 11: *USE DESIGN AIDS*
Find the steel area required to resist a moment $\boldsymbol{M_u}$ of **150 kNm** using a concrete section having $\boldsymbol{b}$ **= 250 mm**, $\boldsymbol{d}$ **= 435 mm**, and $\boldsymbol{h}$ **= 500 mm**. $\boldsymbol{f'_c}$ **= 28 MPa** and $\boldsymbol{f_y}$ **= 420 MPa.**

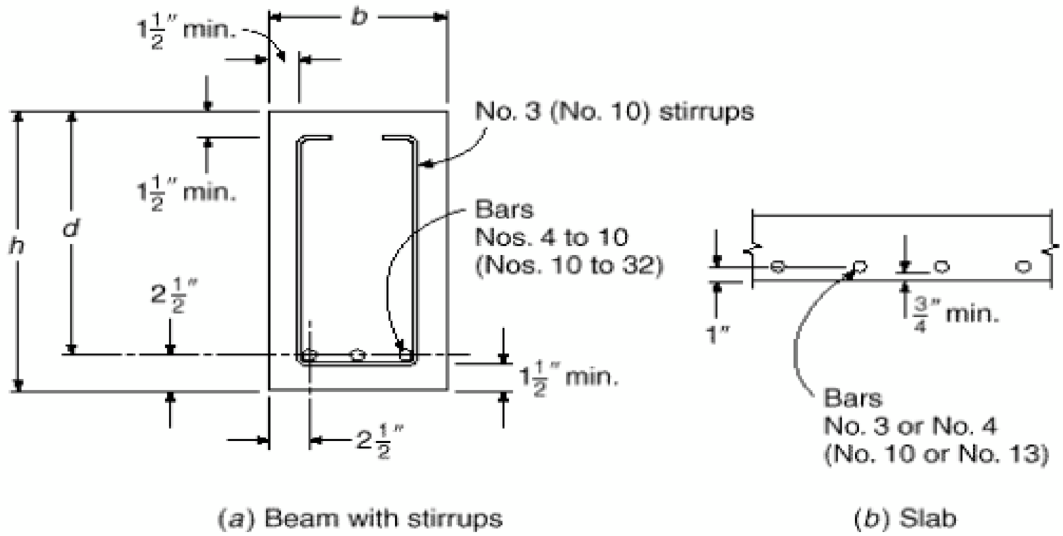
Solution:

$R = M_u / \varphi bd^2 = 3.52$ MPa
Table A.5a, $\rho = 0.0091$ giving $A_s = 0.0091(250)(435) = 990$ mm^2
USE 2 No.25 bars.

Practical Considerations in the Design of Beams

a. Concrete Protection for Reinforcement

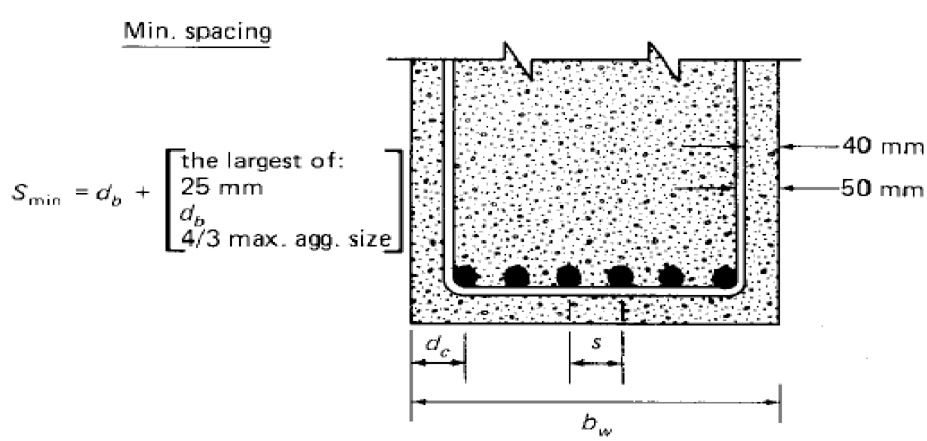
To provide the steel with adequate concrete protection against fire and corrosion, the designer must maintain a certain minimum thickness of concrete cover outside of the outermost steel. The thickness required will vary, depending upon the type of member and conditions of exposure. The requirements of concrete cover in beams and slabs are shown in figure below:



¾ in = 20 mm, 1 in = 25 mm, 1½ in = 40 mm, 2½ in = 65 mm

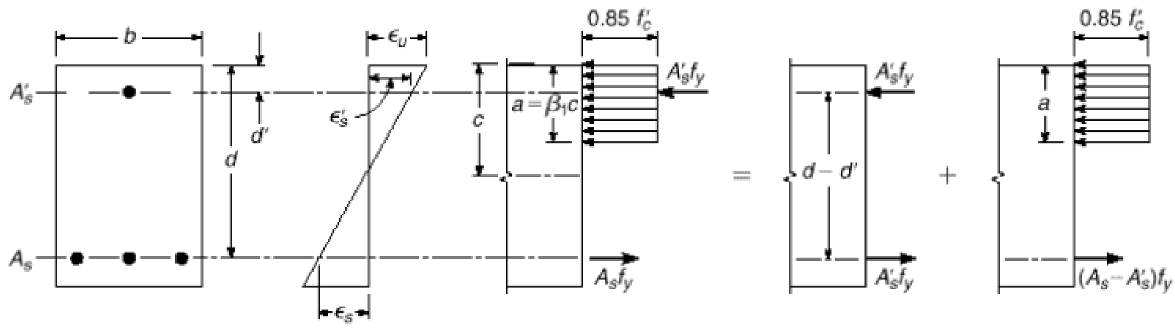
b. Selection of Bars and Bar Spacing

It is often desirable to mix bar sizes to meet steel area requirements more closely. In general, mixed bars should be of comparable diameter, and generally should be arranged symmetrically about the vertical centerline. ACI Code 25.6 specifies that the minimum clear distance between adjacent bars shall not be less than the nominal diameter of the bars or 25 mm. (for columns 1½ bar diameter or 40mm).
Where beam reinforcement is placed in two or more layers, the clear distance between layers must not be less than 25 mm, and the bars in the upper layer should be placed directly above those in the bottom layer. The maximum number of bars that can be placed in a beam of given width is limited by bar diameter, by concrete cover, and by the maximum size of aggregate specified. See Table A.7, and Fig. below.



Rectangular Beams with Tension and Compression Reinforcement

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete cannot develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a ***doubly reinforced*** beam, i.e., with compression as well as tension reinforcement (see Fig.):



(a) (b) (c) (d) (e)

a. Tension and Compression Steel Both at Yield Stress

In a doubly reinforced beam;

If $\rho \le \rho_b$, disregard the compression bars.

If $\rho > \rho_b$, the total resisting moment is the sum of two parts:

The 1st part: $M_{n1} = A'_s f_y (d - d')$

The 2nd part: $M_{n2} = (A_s - A'_s) f_y (d - a/2)$

$$a = (A_s - A'_s) f_y / 0.85 f'_c b$$

$$\rho = A_s / bd \quad \text{and} \quad \rho' = A'_s / bd$$

$$a = (\rho - \rho') f_y d / 0.85 f'_c$$

The total nominal moment:

$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y (d - a/2)$$

The balanced reinforcement ratio for a doubly reinforced beam is $\bar{\rho}_b$:

$$\bar{\rho}_b = \rho_b + \rho'$$

The maximum reinforcement ratio

$$\bar{\rho}_{max} = \rho_{max} + \rho'$$

The maximum reinforcement ratio for $\varphi = 0.9$

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho'$$

b. <u>Compression Steel below Yield Stress</u>

In many cases, the compression bars will below the yield stress at failure ($f'_s < f_y$).

From geometry of the strain diagram, fig. b:

$$c = d'[\varepsilon_u / (\varepsilon_u - \varepsilon_y)]$$

Sum of forces in fig. c gives $\bar{\rho}_{cy}$ **:** the minimum ratio that will ensure yielding of compression steel,

$$\bar{\rho}_{cy} = 0.85\,\beta_1 (f'_c / f_y)(d' / d)[\varepsilon_u / (\varepsilon_u - \varepsilon_y)] + \rho'$$

From figures b and c:

$$\bar{\rho}_b = \rho_b + \rho'(f'_s / f_y)$$

where

$$f'_s = E_s\,\varepsilon'_s = E_s\,[\varepsilon_u - (d' / d)\,(\varepsilon_u + \varepsilon_y)] \leq f_y$$

To determine ρ_{max}, $\varepsilon_t = 0.004$ is substituted for ε_y, giving

$$f'_s = E_s\,\varepsilon'_s = E_s\,[\varepsilon_u - (d' / d)\,(\varepsilon_u + 0.004)] \leq f_y$$

Likewise for $\varepsilon_t = 0.005$

$$f'_s = E_s\,\varepsilon'_s = E_s\,[\varepsilon_u - (d' / d)\,(\varepsilon_u + 0.005)] \leq f_y$$

Hence, the max reinforcement ratio is

$$\bar{\rho}_{max} = \rho_{max} + \rho'(f'_s / f_y)$$

and for $\varphi = 0.9$ is

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho'(f'_s / f_y)$$

if the tensile reinforcement ratio is less than $\bar{\rho}_b$ and less than $\bar{\rho}_{cy}$, then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed:

From strain diagram:

$$f'_s = \varepsilon_u E_s (c - d') / c$$

From Equilibrium:

$$A_s f_y = 0.85 \beta_1 f'_c\, bc + A'_s\, \varepsilon_u E_s (c - d') / c$$

Solve for c, and knowing $a = \beta_1 c$

$$M_n = 0.85\, f'_c\, ab\, (d - a/2) + A'_s f'_s\, (d - d')$$

Example 12: (*Analysis problem, with* $f'_s = f_y$)

A rectangular beam has a width of **300 mm** and an effective depth to the centroid of the tension reinforcement of **600 mm**. The tension reinforcement consists of **six No.32** (#10) bars **in two rows**. Compression reinforcement consisting of **two No.25** (#8) bars is placed **65 mm** from the compression face of the beam. If $f'_c = 35$ **MPa** and $f_y = 420$ **MPa,** what is the design moment capacity of the beam?

Solution:

$A_s = 4914\ mm^2$, $\rho = 4914 / (300\times600) = 0.0273$

$A'_s = 1020\ mm^2$, $\rho' = 1020 / (300\times600) = 0.0057$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded:

$\rho_{max} = 0.85\ \beta_1 (f'_c / f_y)[\ \varepsilon_u / (\varepsilon_u + 0.004)] = 0.0243$ (*or use Table A.4*)

Actual $\rho = 0.0273 > \rho_{max}$, so the beam must be analyzed as a doubly reinforced.

$\bar{\rho}_{cy} = 0.85\ \beta_1 (f'_c / f_y)(d' / d)[\ \varepsilon_u / (\varepsilon_u - \varepsilon_y)] + \rho'$

$= 0.85\times0.8\times(35/420)\times(65/600)\times[0.003/(0.003 - 0.0021)] + 0.0057$

$= 0.0262$

Actual $\rho = 0.0273 > \bar{\rho}_{cy}$, so the compression bars will yield when the beam fails.

$\bar{\rho}_{max} = \rho_{max} + \rho' = 0.0243 + 0.0057 = 0.0300$

Actual $\rho = 0.0273 < \bar{\rho}_{max}$, as required. Then

$a = (A_s - A'_s)f_y / 0.85 f'_c\ b$

$= (4914 - 1020)\ 420 /(0.85\times35\times300) = 183.2$ mm

$c = a / \beta_1 = 183.2 / 0.80 = 229$ mm

$\varepsilon_t = \varepsilon_u(d - c)/c = 0.003\ (600 - 229)/229 = 0.0049$ thus $\varphi = 0.89$

$M_n = M_{n1} + M_{n2} = A'_s f_y\ (d - d') + (A_s - A'_s) f_y\ (d - a/2)$

$= [1020\times420(600 - 65) + 3894\times420\ (600 - 183.2/2)] \times10^{-6} = 1061$ kNm

Design strength is

$\varphi M_n = 0.89 \times 1061 =$ <u>954 kNm</u>

Example 13: (*Design problem, with* $f'_s < f_y$)

A rectangular beam that must carry a service live load of **36 kN/m** and a calculated dead load of **15.3 kN/m** on an **5.5 m** simple span is limited in cross section for architectural reasons to **250 mm** (10 in) width and **500 mm** (20 in) total depth. If If $f'_c = 28$ **MPa** and $f_y = 420$ **MPa,** what steel area(s) must be provided?

Solution:

Factored load, $w_u = 1.2 \times 15.3 + 1.6 \times 36 = 75.96$ kN/m

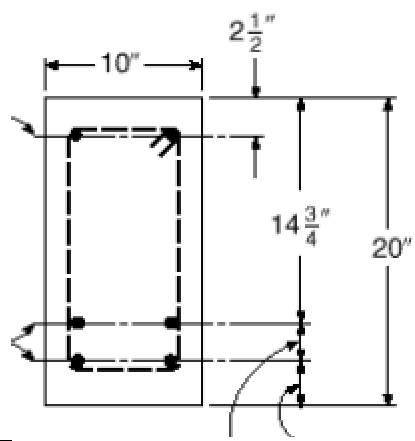
$M_u = 75.96\,(5.5)^2/8 = 287.2$ kNm

Assume tension steel centroid is 100mm above the bottom face and assume compression steel, if required, will be placed 65mm below the top surface. Then ***d*** *= 400mm,* ***d'*** *= 65mm*

Check if the section is singly reinforced:
Table A.4: $\rho_{0.005} = 0.0181$ for $\varphi = 0.9$
$A_s = 250 \times 400 \times 0.0181 = 1810$ mm^2
$a = 1810 \times 420 / 0.85 \times 28 \times 250 = 127.8$ mm
$c = a / \beta_1 = 127.8 / 0.85 = 150.3$ mm
$M_n = A_s f_s (d - a/2) = 1810 \times 420 (400 - 127.8/2) 10^{-6} = 255.5$ kNm
(*Alternatively, Table A.5b, R = 6.39,* $M_n = Rbd^2 = 255.5$ *kNm*)

$\varphi M_n = 230$ kNm < 287.2 kNm, therefore compression steel is needed as well as additional tension steel.

The remaining moment to be carried by compression steel couple:
$M_1 = 287.2 - 230 = 57.2$ kNm
Since $\rho < \bar{\rho}_{cy}$ then compression steel stress $f'_s < f_y$
From strain diagram
$\varepsilon'_s = 0.003\,(150.3 - 65)/150.3 = 0.00170$
$f'_s = \varepsilon'_s E_s = 0.00170 \times 200000 = 340$ MPa
$A'_s = 57.2 \times 10^6 / [340\,(400 - 65)] = 502$ mm^2
Total area of tensile reinforcement at 420 MPa:
$A_s = 1810 + 502(340/420) = 2217$ mm^2
USE **2 No.19 bars** (568 mm^2) as compression reinforcement and **4 No.29 bars** (2580 mm^2) as tension reinforcement. To place tension reinforcement, 2 rows of 2 bars each are used. See figure.



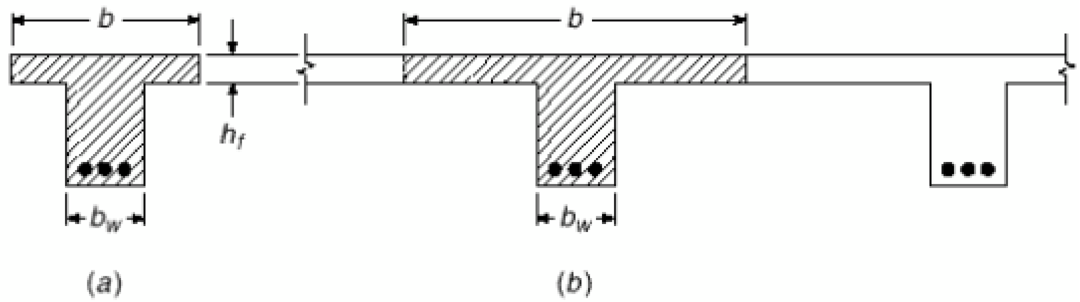
60mm **70mm**

2½ in = 65 mm, 14¾ in = 370 mm, 10 in = 250 mm, 20 in = 500mm.

T Beams

RC floors, roofs, decks, etc., are almost always monolithic. Beam stirrups and bent bars extend up into the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression.
The resulting section is T-shaped rather than rectangular.
The slab forms the beam ***flange***, while the part of the beam projecting below the slab is called the ***web*** or ***stem***.

a. Effective Flange Width



The effective flange width, $\boldsymbol{b}$, has been found to depend on the *beam span* $\boldsymbol{l}$, the *thickness of the slab* $\boldsymbol{h_f}$, and the *clear distance to the next beam* $\boldsymbol{l_c}$.

ACI Code 6.3 requires that:

1. For symmetric T beams:

$\boldsymbol{b \le l/4}$

$\boldsymbol{(b - b_w)/2 \le 8\, h_f}$

$\boldsymbol{(b - b_w)/2 \le l_c/2}$

2. For beams having a slab on one side only:

$\boldsymbol{(b - b_w) \le l/12}$

$\boldsymbol{(b - b_w) \le 6\, h_f}$

$\boldsymbol{(b - b_w) \le l_c/2}$

3. For isolated beams:

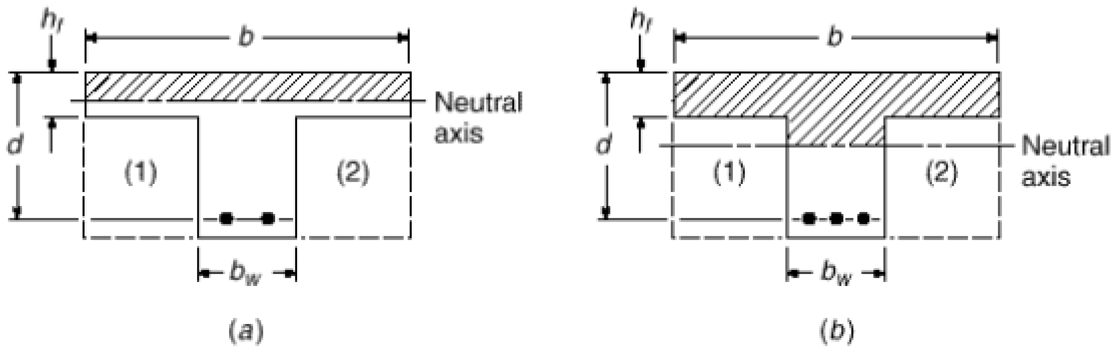
$\boldsymbol{h_f \ge b_w/2}$

$\boldsymbol{b \le 4\, b_w}$

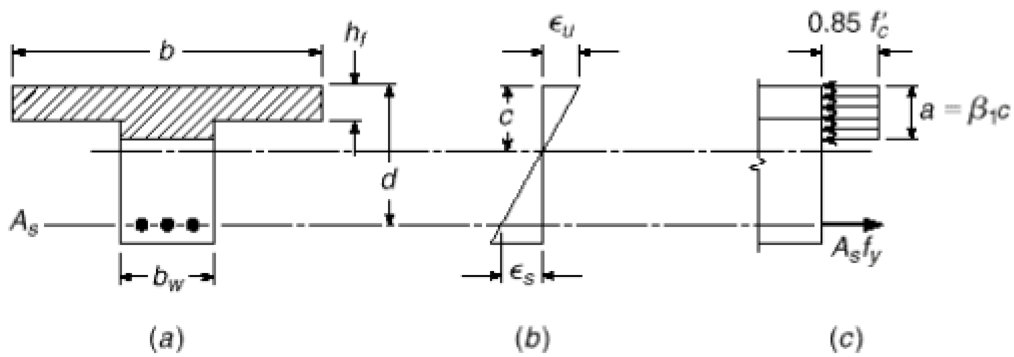
b. Structural Analysis

The neutral axis NA of a T beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials.

If the depth of NA ≤ $\boldsymbol{h_f}$, the beam can be analyzed as if it were a rectangular beam of width $\boldsymbol{b}$. (fig. a). This is because areas (1) and (2) are entirely in tension zone and thus disregarded in flexural calculations.



When the NA is in the web (fig. b), method is developed to account for the actual T-shaped compressive zone.



It is convenient to divide the total tensile steel into two parts. The first part, $\boldsymbol{A_{sf}}$, represents the steel area required to balance the compressive force in the overhanging portions.

$$\boldsymbol{A_{sf} = 0.85\, f'_c (b - b_w) h_f / f_y}$$

$$\boldsymbol{M_{n1} = A_{sf}\, f_y (d - h_f/2)}$$

The remaining steel area, $(\boldsymbol{A_s} - \boldsymbol{A_{sf}})$, is balanced by the compression in the rectangular portion of the beam. In this zone:

$$\boldsymbol{a = (A_s - A_{sf})\, f_y / (0.85\, f'_c\, b_w)}$$

$$\boldsymbol{M_{n2} = (A_s - A_{sf})\, f_y\, (d - a/2)}$$

The total nominal resisting moment is the sum of the parts:

$$\boldsymbol{M_n = M_{n1} + M_{n2} = A_{sf}\, f_y (d - h_f/2) + (A_s - A_{sf})\, f_y\, (d - a/2)}$$

From strain diagram:

$$c/d_t \leq \varepsilon_u/(\varepsilon_u + \varepsilon_t)$$

Setting $\varepsilon_u = 0.003$ and $\varepsilon_t = 0.004$ provides a max $c/d_t = 0.429$
The condition of tensile steel yield to occur prior to concrete crushing is satisfied if

$$\rho_{w,max} = \rho_{max} + \rho_f$$

where

$\rho_w = A_s / b_w d$ *and* $\rho_f = A_{sf} / b_w d$

The minimum reinforcement is based on b_w:

$$\rho_{min} = 0.25\sqrt{f'_c}/f_y \geq 1.4/f_y$$
$$A_{s,min} = \rho_{min}\, b_w d$$

c. Examples of T Beams

Example 14: (*Analysis problem*)
An isolated T beam is composed of a flange **700 mm** wide and **150 mm** deep cast monolithically with a web of **250 mm** width that extends **600 mm** below the bottom surface of the flange to produce a beam of **750 mm** total depth. Tensile reinforcement consists of **6 No.32** (#10) bars placed in two horizontal rows. The centroid of the bar group is **650 mm** from the top of the beam. If $f'_c = 21$ **MPa** and $f_y = 420$ **MPa**, what is the design moment capacity of the beam?

Solution:
Check flange dimensions, $h_f \geq b_w/2$: 150 > 250/2 , 150 > 125
$b \leq 4\, b_w$: 700 < 4×250 , 700 < 1000 OK
Area of 6No.32 = 4914 mm^2
Check NA location:
$a = A_s f_y/(0.85 f'_c b)$= 4914×420/0.85×21×700 = 165.2 mm > h_f = 150mm
so T beam analysis is required.

$A_{sf} = 0.85 f'_c(b - b_w)h_f / f_y$ = 0.85×21(700 – 250)×150 /420 = 2869 mm^2
$M_{n1} = A_{sf} f_y(d - h_f/2)$ = 2860×420 (650 – 150/2)×10^{-6} = 692.8 kNm

$a = (A_s - A_{sf}) f_y / (0.85 f'_c b_w)$ = 2045×420/(0.85×21×250) = 192.5 mm
$M_{n2} = (A_s - A_{sf}) f_y (d - a/2)$=2045×420 (650 – 192.5/2)×10^{-6}=475.6 kNm

$c = a / \beta_1$ = 192.5 / 0.85 = 226.5 mm
d_t = 685 mm to the lowest bar,
c / d_t = 226.5 / 685 = 0.331 < 0.375, so the ε_t > 0.005 and φ = 0.9
φM_n = 0.9 (692.9 + 475.6) = 1052 kNm

Example 15: (*Design problem*)

A floor system consists of a **75 mm** concrete slab supported by continuous T beams with a **7.5 m** span, **1.2 m** on centers. Web dimensions are b_w = **275 mm** and d = **500 mm**. What tensile steel area is required at midspan to resist a factored moment of **725 kNm** if f_y = **420 MPa** and f'_c = **21 MPa**?

Solution:

Determine effective flange width:
$b = 16\, h_f + b_w$ = 16×75 + 275 = 1475 mm = 1.475 m
$b = l / 4$ = 7.5 / 4 = 1.875 m
b = 1.2 m (c. / c. spacing)
The controlling b = 1.2 m

Assume $a = h_f$ = 75 mm
$d - a/2$ = 500 – 75/2 = 462.5 mm
Trial:
$A_s = M_u / \varphi f_y (d - a/2)$ = 725×10^6 /[0.9×420×462.5] = 4147 mm^2
Check $a = A_s f_y /(0.85 f'_c b)$= 4147×420/[0.85×21×1200] = 81.3 mm > h_f
T beam design is required and φ = 0.9 is assumed.

$A_{sf} = 0.85 f'_c(b - b_w)h_f / f_y$ = 0.85×21(1200 – 250)×75 /420 = 2948 mm^2
$\varphi M_{n1} = \varphi A_{sf} f_y(d - h_f /2)$ = 0.9[2948×420 (500 – 75/2)×10^{-6} = 515.4 kNm
$\varphi M_{n2} = M_n - \varphi M_{n1}$ = 725 – 515.4 = 209.6 kNm

Assume a = 100 mm
$A_s - A_{sf} = \varphi M_{n2} / \varphi f_y (d - a/2)$= 209.6 ×106 /0.9×420×(500 – 50) = 1232 mm^2
Check $a = (A_s - A_{sf}) f_y /(0.85 f'_c b_w)$= 1232×420/[0.85×21×275] = 105.4 mm
This is close to 100 mm assumed. OK
$A_s = A_{sf} + A_s - A_{sf}$ = 2948 + 1232 = 4181 mm^2
Check
$c = a / \beta_1$ = 105.4 / 0.85 = 124 mm
c / d_t = 124 / 500 = 0.248 < 0.375, so the ε_t > 0.005 and φ = 0.9
Design is satisfactory.